CHAPTER 10

Data Analysis and Probability

Section 10A

Data Analysis

- Connecting Algebra to Data Analysis Bar and Circle Graphs
- 10-1 Organizing and Displaying Data
- 10-2 Frequency and Histograms
- 10-3 Data Distributions
- Extension Dot Plots and Distributions
- Technology Lab Use Technology to Make Graphs
- 10-4 Misleading Graphs and Statistics
- Connecting Algebra to Data Analysis Sampling and Bias

Section 10B

Probability

- Algebra Lab Simulations
- 10-5 Experimental Probability
- Technology Lab Use Random Numbers
- 10-6 Theoretical Probability
- 10-7 Independent and Dependent Events
- Algebra Lab Compound Events

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 10

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Connecting Algebra to Data Analysis	10-1 Lesson	10-2 Lesson	10-3 Lesson	10-3 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
Extension	Technology Lab 10-4 Lesson	10-4 Lesson Connecting Algebra to Data Analysis	Connecting Algebra to Data Analysis Ready to Go On?	Focus on Problem Solving Algebra Lab
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
10-5 Lesson	10-5 Lesson Technology Lab	10-6 Lesson	10-6 Lesson 10-7 Lesson	10-7 Lesson
DAY 16	**DAY 17**	**DAY 18**	**DAY 19**	
Algebra Lab	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test	

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 10

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Connecting Algebra to Data Analysis 10-1 Lesson	10-2 Lesson 10-3 Lesson	10-3 Lesson Extension	Technology Lab 10-4 Lesson Connecting Algebra to Data Analysis	Connecting Algebra to Data Analysis Multi-Step Test Prep Ready to Go On? Algebra Lab
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
10-5 Lesson Technology Lab	10-6 Lesson 10-7 Lesson	10-7 Lesson Algebra Lab	Multi-Step Test Prep Ready to Go On? Chapter Review	Chapter Test

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 10

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

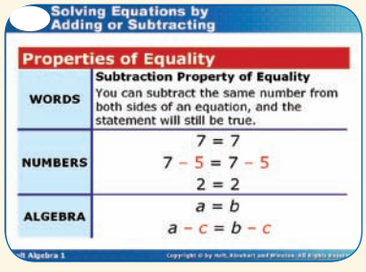
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

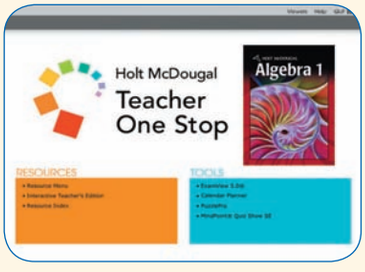
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 10

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

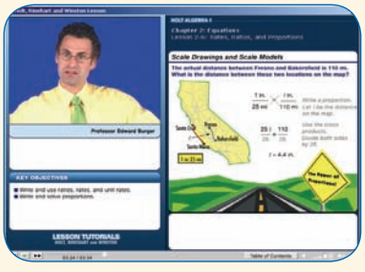
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

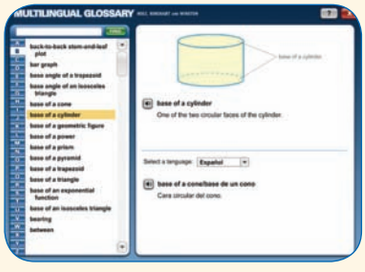
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



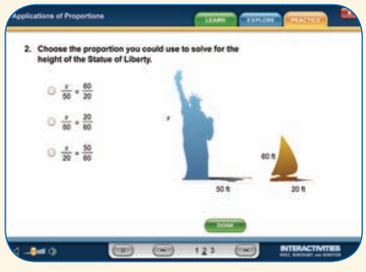
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 10

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
***IDEA Works!*® Modified Worksheets and Tests**

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
***IDEA Works!*® Modified Worksheets and Tests**

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
***ExamView*™ Assessment Suite**

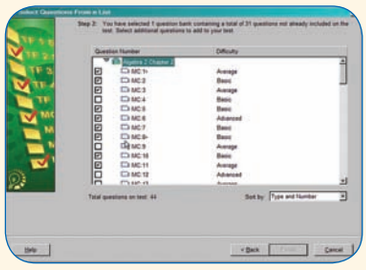
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
***ExamView*™ Assessment Suite**
***IDEA Works!*® Modified Worksheets and Tests**

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 10

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

MULTIPLE CHOICE

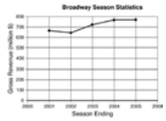
A Chapter Test

C Chapter Test

B Chapter Test

Select the best answer.

1. Use this line graph to identify the period that saw the greatest change in gross revenue.



A 2001–2002 C 2003–2004
B 2002–2003 D 2004–2005

2. Which type of graph would be best for displaying this data?

Actors/Actresses with The Most Oscar Nominations

Actor/Actress	Nominations
Meryl Streep	13
Katherine Hepburn	12
Jack Nicholson	12
Bette Davis	10
Laurence Olivier	10

F bar graph H histogram
G circle graph J line graph

3. This stem-and-leaf plot gives the number of gold medals won by ten countries during the 2004 Olympics. Which data set is plotted?

Stem	Leaves
0	9 9
1	0 1 4 6 7
2	7
3	2 5

Key: 3 | 2 means 32

A {0, 1, 2, 4, 5, 6, 7, 7, 9, 9}
B {1, 11, 23, 41, 53, 61, 71, 72, 90, 90}
C {9, 9, 10, 11, 14, 16, 17, 27, 32, 35}
D {27, 99, 325, 101, 467}

The ages of the U.S. Presidents that were inaugurated during the 1900's are given below. Use this data for questions 4–7.

Ages at Inauguration
42 51 56 55 51 54 51 60 62 43 55 56 61 52 69 64 46

4. Which frequency table reflects the data?

F

Ages	Freq.
41–50	3
51–60	4
61–70	5

H

Ages	Freq.
41–50	3
51–60	10
61–70	4

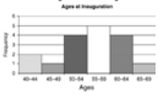
G

Ages	Freq.
41–50	3
51–60	9
61–70	5

J

Ages	Freq.
41–50	6
51–60	8
61–70	3

5. What is wrong with this histogram?



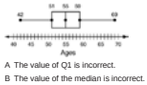
A The bar for 40–44 is too short and the bar for 45–49 is too tall.
B The bar for 50–54 is too tall.
C The bar for 50–54 is too short and the bar for 55–59 is too tall.
D The bar for 55–59 is too short.

6. Find the mean, median, and mode. (Round answers to the nearest tenth.)

	mean	median	mode
F	54.6	55	51
G	54.6	62	51
H	55	54.6	no mode
J	55	55	no mode

B Chapter Test *(continued)*

7. What, if anything, is wrong with this box-and-whisker plot?



A The value of Q1 is incorrect.
B The value of the median is incorrect.
C The value of Q3 is incorrect.
D The boxplot is correct.

8. This graph shows how Russell spends his income each month. Explain why the graph is misleading.

F The entertainment sector is too small relative to the groceries sector.
G The rent sector is too large relative to the entertainment sector.
H The sample size is too small.
J The sectors do not add to 100%.

An experiment consists of spinning a spinner. Use these results for questions 9 and 10.

Outcome	Freq.
Red	4
Green	10
Blue	6

9. What is the experimental probability that the spinner lands on red?
A 4% C 25%
B 20% D 40%

10. If Angie spins the spinner 250 times, predict the number of times it will land on green.
F 10 H 50
G 25 J 125

11. Find the theoretical probability of randomly choosing a vowel from the letters in EXPERIMENT.
A $\frac{1}{5}$ C $\frac{2}{5}$
B $\frac{3}{10}$ D $\frac{3}{5}$

12. The probability of picking a red marble from a bag is $\frac{2}{7}$. What are the odds against picking a red marble?
F 2:5 H 5:7
G 5:2 J 7:5

13. A number cube is rolled 2 times in a row. What is the probability of rolling a multiple of 3 both times?
A $\frac{1}{36}$ C $\frac{1}{15}$
B $\frac{1}{18}$ D $\frac{1}{9}$

14. A game board has 8 cards, and 2 say WIN. Mayela picks 2 cards without replacing the first. What is the probability that neither say WIN?
F $\frac{15}{32}$ H $\frac{9}{16}$
G $\frac{15}{28}$ J $\frac{9}{14}$

15. Which are independent events?
A Two number cubes are rolled.
B A coin is drawn from a jar, set aside, and then another coin is drawn.
C Two freshmen are chosen for a class project.
D 20 students are selected for the baseball team.

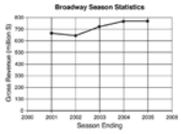
FREE RESPONSE

A Chapter Test

C Chapter Test

B Chapter Test

1. Use this line graph to identify the period that saw the least change in gross revenue.



2. Use this data to make a graph. Explain why you chose that type of graph.

Mother Tongues for the Population of Quebec, 2001

Language	People (thousands)
English only	572
French only	5789
English and French	55
Other	709

3. The number of gold medals won by ten countries during the 1996 Olympics are given below. Use the data to make a stem-and-leaf plot.
{9, 16, 9, 15, 20, 13, 7, 26, 9, 44}

The ages of the Board of Directors of an insurance company are given below. Use this data for questions 4–7.

Ages of Board of Directors
67 52 54 57 61 56 61 69 64 63 57 60 50

4. Complete this frequency table.

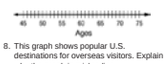
Ages	Frequency
50–54	
55–59	
60–64	
65–64	

5. Use your frequency table in question 4 to make a histogram.

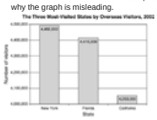
6. Find the mean, median, and mode. (Round answers to the nearest tenth.)
mean: _____
median: _____
mode: _____

B Chapter Test *(continued)*

7. Use the data to make a box-and-whisker plot.



8. This graph shows popular U.S. destinations for overseas visitors. Explain why the graph is misleading.



An experiment consists of spinning a spinner. Use these results for questions 9 and 10.

Outcome	Frequency
Red	6
Blue	9
Green	15

9. What is the experimental probability that the spinner lands on red?

10. If Rhonda spins the spinner 400 times, predict the number of times it will land on blue.

11. Find the theoretical probability of randomly choosing a vowel from the letters in EVENTS.

12. The probability of picking a red marble from a bag is $\frac{3}{10}$. What are the odds against picking a red marble?

13. A number cube is rolled 2 times in a row. What is the probability of rolling a number greater than 2 both times?

14. A game board has 9 cards, and 3 say WIN. Michal picks 2 cards without replacing the first. What is the probability that neither say WIN?

15. Is the event dependent or independent? Three students are selected at random to participate in a demonstration.

PERFORMANCE ASSESSMENT

Chapter Test

Hangman

Purpose

This performance task assesses the student's ability to solve problems involving the theoretical probability of an event.

Time

30–55 minutes

Grouping

Pairs

Preparation Hints

Divide the students into pairs and have them play *Hangman* using vocabulary from the chapter. After one or two rounds, ask whether anyone won a game with no wrong letters or lost a game with no correct letters. Ask students how likely they think these two situations are.

Overview

In this activity, students find the theoretical probabilities of winning by guessing all of the correct letters and losing by guessing all incorrect letters. The activity concludes with a game show scenario in which frequently used letters are automatically given.

Introduce the Task

Distribute the activity and review the scenario outlined in the opening paragraphs. Make sure students understand that 7 correct letters are required to win and 6 wrong letters are required to lose. Also emphasize that the activity calculates theoretical probabilities based on random guessing; for simplicity the influences of strategy and skill are ignored.

Performance Indicators

_____ Determines theoretical probabilities by finding the ratio of favorable outcomes to total outcomes.

_____ Recognizes an impossible event and understands the probability is 0.

Scoring Rubric

Level 4: Student solves problems correctly and gives good explanations.

Level 3: Student solves problems but does not give satisfactory explanations.

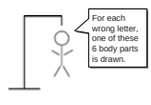
Level 2: Student solves some problems but does not give satisfactory explanations.

Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

Hangman

In the game called *Hangman*, your opponent thinks of a mystery word and draws blanks to represent each letter in the word. You then try to fill in the blanks by guessing letters, but the order of your guesses does not matter. For each correct guess, your opponent fills the letter into all of the blanks where it belongs. For each wrong guess, your opponent draws part of a hangman diagram. In one variation of the game, you lose after guessing 6 wrong letters.



_ _ _ _ _ _ _ _ _

For 1–6, assume that the mystery word is UNLIKELY. Also assume that you randomly guess letters. (That is, you don't use any strategies such as guessing vowels first.)

1. Is selecting a letter in *Hangman* is the event dependent or independent? Explain.
2. If a computer program chooses the letters at random and can choose the same letters more than once, what is the probability the program will win the game?
3. What is the *probability* that you correctly guess all 7 letters without a single wrong guess?
4. What is the probability that the first three guesses are correct?
5. What is the probability that the first two guesses are incorrect?
6. The bonus round of a popular television game show is similar to *Hangman* except the letters R, S, T, L, N, and E are automatically revealed. You get to guess 3 more consonants and 1 more vowel. What is the probability of correctly guessing all of the remaining letters? Support your answer.

CHAPTER 10

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

SECTION 10A
Data Analysis

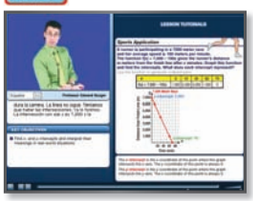
MULTI-STEP TEST PREP Students use graphs and statistics to model real-world situations at the Olympics.

SECTION 10B
Probability

MULTI-STEP TEST PREP Students use probability to model real-world games.

Interactivities Online ▶

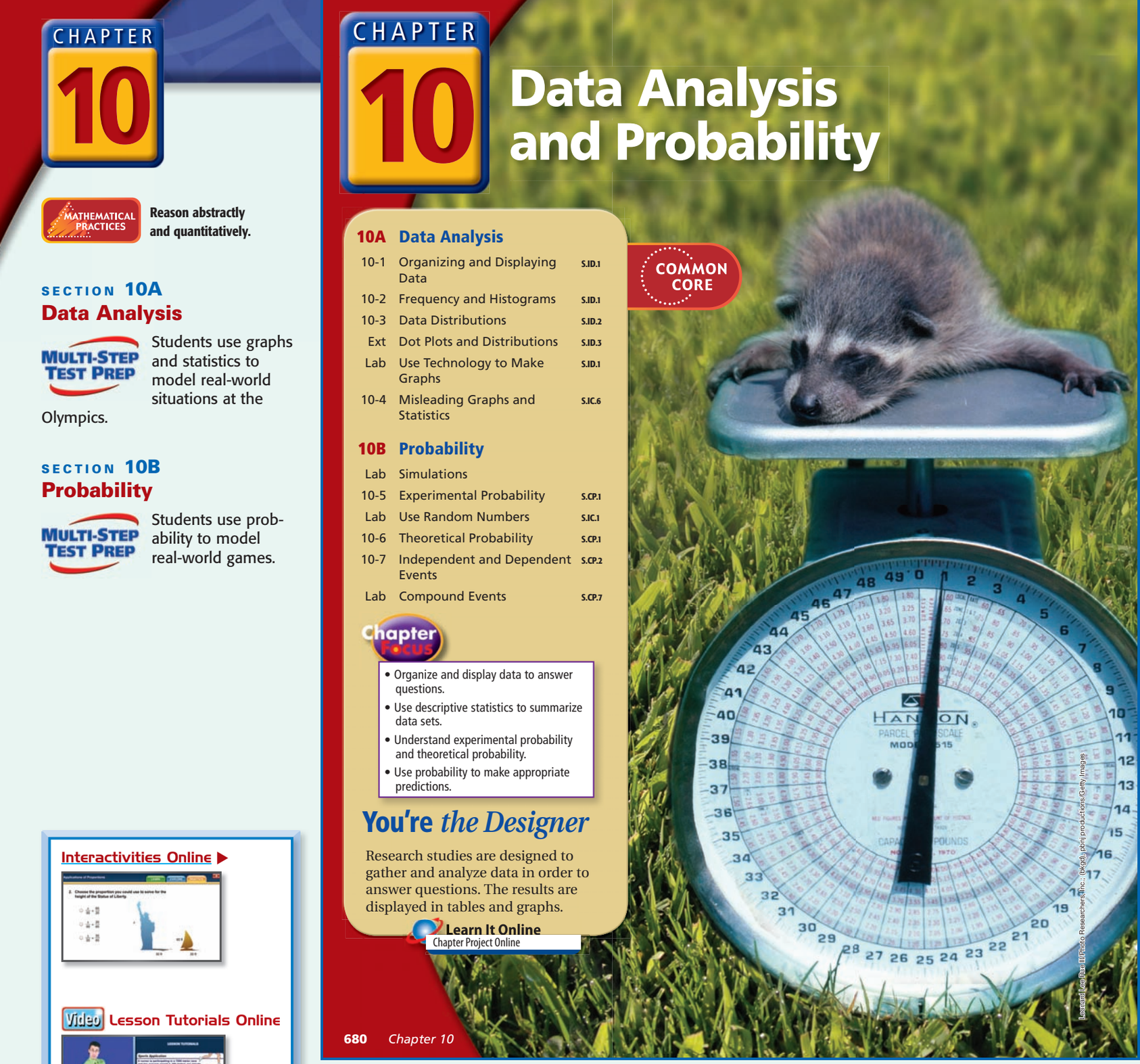
Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 10

Data Analysis and Probability



COMMON CORE

10A Data Analysis

10-1	Organizing and Displaying Data	S.ID.1
10-2	Frequency and Histograms	S.ID.1
10-3	Data Distributions	S.ID.2
Ext	Dot Plots and Distributions	S.ID.3
Lab	Use Technology to Make Graphs	S.ID.1
10-4	Misleading Graphs and Statistics	S.IC.6

10B Probability

Lab	Simulations	
10-5	Experimental Probability	S.CP.1
Lab	Use Random Numbers	S.IC.1
10-6	Theoretical Probability	S.CP.1
10-7	Independent and Dependent Events	S.CP.2
Lab	Compound Events	S.CP.7

Chapter Focus

- Organize and display data to answer questions.
- Use descriptive statistics to summarize data sets.
- Understand experimental probability and theoretical probability.
- Use probability to make appropriate predictions.

You're *the Designer*

Research studies are designed to gather and analyze data in order to answer questions. The results are displayed in tables and graphs.

Learn It Online
Chapter Project Online

Leonard Lee Rue III/Photo Researchers, Inc.; (bkgd) pdnj productions/Getty Images

Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online

680 Chapter 10

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. difference **D**
2. factor **B**
3. natural numbers **F**
4. ratio **E**
5. sum **A**

A. the result of an addition

B. a whole number that is multiplied by another whole number to get a product

C. numbers that can be expressed in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$

D. the result of a subtraction

E. a comparison of two quantities by division

F. the counting numbers: 1, 2, 3, …

Solve Proportions

Solve each proportion.

6. $\frac{3}{4} = \frac{x}{12}$ **9**
7. $\frac{15}{9} = \frac{3}{x}$ **1.8**
8. $\frac{10}{20} = \frac{x}{100}$ **50**
9. $\frac{250}{1500} = \frac{x}{100}$ **$16\frac{2}{3}$**

Compare and Order Real Numbers

Compare. Write <, >, or =.

10. 20 **>** 13
11. $\frac{2}{3}$ **>** $\frac{1}{2}$
12. $\frac{3}{4}$ **<** $\frac{7}{9}$
13. 0.75 **=** $\frac{9}{12}$

Order the numbers from least to greatest.

14. $\frac{1}{2}, \frac{4}{5}, \frac{1}{8}, \frac{3}{4}, \frac{2}{3}$ **$\frac{1}{8}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$**
15. $0.12, \frac{2}{5}, \frac{3}{4}, 0.3, \frac{1}{3}$ **$0.12, 0.3, \frac{1}{3}, \frac{2}{5}, \frac{3}{4}$**

Multiply Decimals

Multiply.

16. 0.25×300 **75**
17. 0.5×4000 **2000**
18. 0.05×200 **10**
19. 0.125×9600 **1200**

Divide Decimals

Divide.

20. $435 \div 10$ **43.5**
21. $32 \div 100$ **0.32**
22. $777 \div 1000$ **0.777**
23. $295 \div 10{,}000$ **0.0295**

Fractions, Decimals, and Percents

Write the equivalent decimal.

24. $\frac{3}{5}$ **0.6**
25. 45% **0.45**
26. $\frac{3}{4}$ **0.75**
27. 8% **0.08**

Write the equivalent percent.

28. $\frac{1}{4}$ **25%**
29. 0.2 **20%**
30. 0.36 **36%**
31. $\frac{1}{10}$ **10%**

CHAPTER 10

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

***Are You Ready? Intervention and Enrichment* Worksheets**

***Are You Ready?* Online**

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention,* Chapter 10 Worksheets	*Are You Ready? Intervention,* Chapter 10 Online	
Solve Proportions	Skill 77	Activity 77	Diagnose and Prescribe Online
Compare and Order Real Numbers	Skill 16	Activity 16	
Multiply Decimals	Skill 45	Activity 45	
Divide Decimals	Skill 46	Activity 46	
Fractions, Decimals, and Percents	Skill 14	Activity 14	

***Are You Ready? Enrichment,* Chapter 10**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. the middle value that divides a set of data into 2 equal parts
2. Quarter, quarterly, quartet; they all refer to something with 4 parts.
3. Experimental probability describes the likelihood that something will happen based on trials and experiences. Theoretical probability describes the likelihood that something will happen based on formulas and theorems.

CHAPTER 10 Study Guide: Preview

Where You've Been

Previously, you

- read information from tables and graphs.
- added, subtracted, multiplied, and divided real numbers.
- worked with ratios and percents.

In This Chapter

You will study

- how to organize data in tables, graphs, and plots.
- how to find the central tendency of a data set by calculating mean, median, and mode.
- writing experimental and theoretical probability as ratios, percents, and decimals.

Where You're Going

You can use the skills in this chapter

- to present your findings from science laboratory experiments in an appropriate and accurate graphical form.
- to be more informed about statistical information in the news and not to be misled by how it is presented.

Key Vocabulary/Vocabulario

dependent events	sucesos dependientes
experimental probability	probabilidad experimental
frequency	frecuencia
independent events	sucesos independientes
median	mediana
outlier	valor extremo
probability	probabilidad
quartile	cuartil
theoretical probability	probabilidad teórica

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The *median strip* is the middle region that divides a highway in half. Use this knowledge to define **median** as it relates to a set of data.
2. The word **quartile** starts with the prefix *quart-*. What are some other words that start with the prefix *quart-*? What do they all have in common?
3. **Probability** is the chance something will happen. Based on your understanding of the words *experiment* and *theory*, compare and contrast the terms **experimental probability** and **theoretical probability**.

MATHEMATICAL PRACTICES

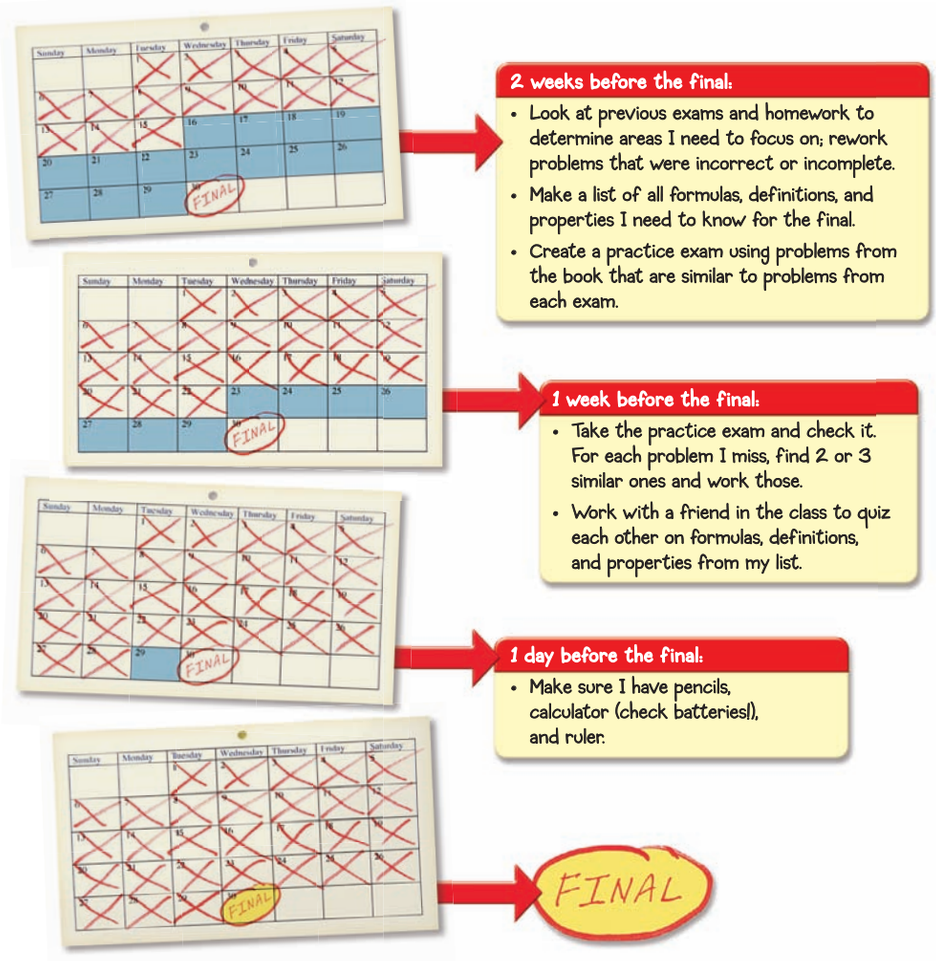
The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 706–709, 718–721, 724, 730–732, 738–739, 743, 744–747, 750
2. **Reason abstractly and quantitatively.** Pages 694, 698–700, 705, 706–707, 731, 745
3. **Construct viable arguments and critique the reasoning of others.** Pages 690, 691–694, 697, 699–700, 705, 708, 717, 720, 729, 731, 737, 738–739, 744, 746
4. **Model with mathematics.** Pages 709, 741, 742, 747
5. **Use appropriate tools strategically.** Pages 693, 714, 726, 733
6. **Attend to precision.** Pages 691–692, 707, 731
7. **Look for and make use of structure.** Pages 701, 741
8. **Look for and express regularity in repeated reasoning.** Page 748

682 Chapter 10

Study Strategy: Prepare for Your Final Exam

Math is a cumulative subject, so your final exam will probably cover all of the material you have learned since the beginning of the course. Preparation is essential for you to be successful on your final exam. It may help you to make a study timeline like the one below.



Try This

1. Create a timeline that you will use to study for your final exam.

Standards for Mathematical Content High School

		LESSON
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA Summarize, represent, and interpret data on a single count or measurement variable		
CC.9-12.S.ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	10-1, 10-2, 10-3, 10-3 Ext, TLab 10-3
CC.9-12.S.ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	10-3
CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY Understand independence and co probability and use them to inter		
CC.9-12.S.IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	TLab 10-5
~~CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY~~ Understand independence and conditional probability and use them to interpret data		
CC.9-12.S.CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	10-5, 10-6, 10-7
CC.9-12.S.CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	10-7

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE Online Edition

Study Strategy: Prepare for Your Final Exam

Discuss For the two weeks before the final exam, students can place each of the chapter numbers into a block on a calendar, using those days to review previous exams, homework, formulas, properties, and vocabulary from that chapter.

Extend Have students work in small groups to list strategies they have used to study for exams in this or other courses. Students can make posters describing their most effective strategies. Display the posters in the weeks leading up to the final exam.

Answer to *Try This*

1. Check students' work.

Reading Connection

The Great Equations
by Robert P. Crease

From $1 + 1 = 2$ to $E = mc^2$, several equations stand out in the history of mathematics and science. The author discusses ten such equations, making clear that each was not the result of a sudden brainstorm, but instead was the long, painstaking work of a succession of thinkers. Newton's Law of Universal Gravitation is an example of the "inverse-square" relationship, which has countless applications in science and industry.

Activity After students read Chapter 2, ask them to explain how the equation describing the force of gravity illustrates both direct and inverse variation. The force of gravity varies directly with the product of the masses of two objects and inversely with the square of the distance between them.

SECTION 10A

Data Analysis

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 10-1 Organizing and Displaying Data • Organize data in tables and graphs. • Choose a table or graph to display data. ☐ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** magazines, newspapers
Lesson 10-2 Frequency and Histograms • Create stem-and-leaf plots. • Create frequency tables and histograms. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		**Optional** coins (MK), sticky notes
Lesson 10-3 Data Distributions • Describe the central tendency of a data set. • Create and interpret box-and-whisker plots. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** colored pencils (MK)
Extension Dot Plots and Distributions • Create dot plots. • Use a dot plot to describe the shape of a data distribution. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		
Technology Lab Use Technology to Make Graphs • Use a spreadsheet or graphing calculator to create graphs. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** spreadsheet program, graphing calculator
Lesson 10-4 Misleading Graphs and Statistics • Recognize misleading graphs. • Recognize misleading statistics. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 31, 33, 35a, 35b, 37, 41

Lesson 2: 1, 3, 5, 7, 9, 11a, 11b, 15a, 19, 21

Lesson 3: 3, 5, 7, 9, 13, 15, 17, 19, 21, 25, 27, 29, 31, 33, 37, 39, 41, 43, 49, 51, 53, 55, 57

Lesson 4: 3a, 3b, 5, 7a, 7b, 9a, 9b, 15

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

MEAN, MEDIAN, AND MODE

Lesson 10-3

The *measures of central tendency,* mean, median, and mode, are three ways of summarizing a data set by using a single value. With this goal in mind, students may wonder which measure best represents a particular set of data. Although there is no definitive answer to this question, there are cases in which one measure is clearly more effective at describing a data set than the others. Some general guidelines for choosing a measure to describe a data set can be established.

Mean: The mean (or average) takes every data value into account, is easy to calculate, and works well for describing data sets that are normally distributed. (In a data set that is normally distributed, the graph of the distribution is a bell-shaped curve with the mean at the center.) The mean is not as useful for sets that contain outliers because the outliers can have a large effect on the mean.

Median: The median is also easy to calculate. The median is useful for describing data sets that are not normally distributed because it is much less affected by outliers than the mean.

Mode: The mode is useful when the frequency of data values is important or when the data cluster around multiple values. Among the mean, median, and mode, only the mode can be used to summarize a set of non-numerical data, such as favorite colors.

Each measure provides a slightly different perspective on a data set. The clearest understanding of a data set is generally obtained when all three measures are considered as a group.

Students should recognize that the mean of a data set need not be one of the data values. The same is true of the median. For example, the data set {10, 20} has a mean and median of 15. On the other hand, the mode, if it exists, must be one of the values in the data set.

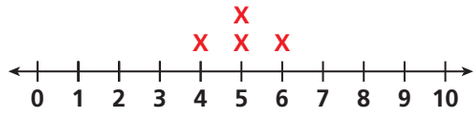
VARIABILITY

Lesson 10-3

The mean, median, and mode are all ways to describe the "typical" value in a data set. It is also useful to describe the *variability,* or spread, of a data set. The simplest way to quantify variability is with the range.

The *range* of a data set is the difference between the greatest and least values in the set. Consider the two data sets shown below. Both have a mean, median, and mode of 5. However, the values in data set *B* are more spread out, and this is reflected in the fact that the data set has a greater range. The range of data set *B* is 6, as compared to a range of 2 for data set *A*.

Data Set *A*



Data Set *B*

In future math courses, students will learn more sophisticated ways to measure variability. Two of the most useful measures in statistics are the variance and the standard deviation.

The *variance,* denoted σ^2, is the average square deviation of each value from the mean. For data set *A*, the variance is

$$\sigma^2 = \frac{(4-5)^2 + (5-5)^2 + (5-5)^2 + (6-5)^2}{4} = 0.5.$$

For data set *B*, the variance is

$$\sigma^2 = \frac{(2-5)^2 + (5-5)^2 + (5-5)^2 + (8-5)^2}{4} = 4.5.$$

The *standard deviation,* σ, is the square root of the variance. For data set *A,* the standard deviation is $\sigma = \sqrt{0.5} \approx 0.7$. For data set *B,* the standard deviation is $\sigma = \sqrt{4.5} \approx 2.1$.

As the examples show, a greater variance and a greater standard deviation indicate a greater spread of data.

Organizer

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Students use data presented in bar graphs and circle graphs to solve equations.

Online Edition

Countdown

Teach

Remember

Students review and apply writing equations.

Teaching Tip **Visual Cues** Direct students' attention to the bar graph in **Example 1.** Point out that the top of the second bar is missing because it has been torn off. Ask for ideas for how the missing information could be found.

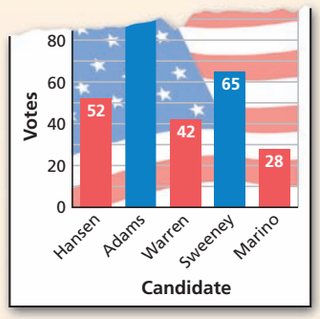
Learn It Online
State Resources Online

Bar and Circle Graphs

Data displayed in bar graphs and circle graphs can be used to solve equations. In these problems, parts of the graphs are missing.

Example 1

The top part of this graph was torn off. If Warren received 15% of the votes, how many votes did Adams receive?



Step 1 Find the total number of votes. Let t represent the total.

15%	of	the total votes	is	42 votes.
0.15	•	t	=	42

$0.15t = 42$

$t = 280$ votes

Step 2 Find the number of votes Adams received.

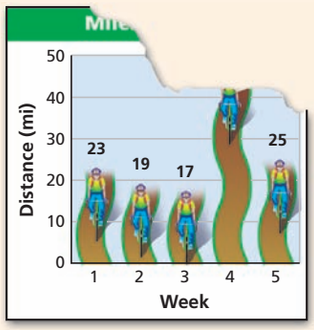
Let a represent the number of votes received by Adams. Let h, w, s, and m represent the number of votes received by Hansen, Warren, Sweeney, and Marino.

$t = a + h + w + s + m$
$280 = a + 52 + 42 + 65 + 28$ *Substitute the numbers shown on the graph.*
$280 = a + 187$ *Simplify the right side of the equation.*
$-187 \quad -187$ *Subtract 187 from both sides.*
$93 = a$

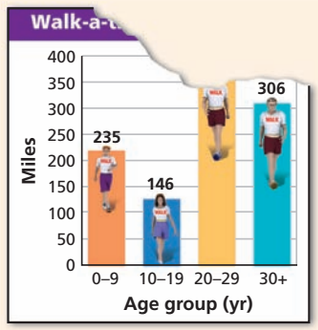
Adams received 93 votes.

Try This

1. The missing bar is twice as tall as the bar for week 2. How many total miles did Kim bike in these five weeks? **122 mi**



2. People aged 20–29 years walked 275 more miles than the oldest age group. Find the total miles walked by all age groups. **1268 mi**



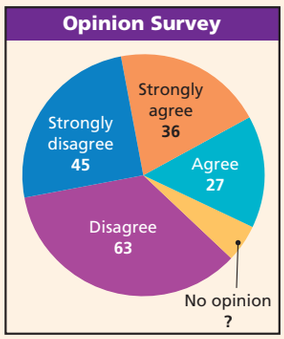
Remember that a circle graph represents all the data in a data set. The percent represented by each section is a part of the whole data set, so the sum of all the percents must be 100%.

Example 2

A survey asked people in a neighborhood to agree or disagree with the following statement:

"We need a traffic light at Jefferson Avenue and Third Street."

If 35% of the people disagreed with the statement, how many people had no opinion?



The number of people who answered "no opinion" is missing from the graph.

Step 1 Find the total number of people who answered the survey. Let t represent the total number of people.

35%	of	the total number of people	is	63 people.
0.35	•	t	=	63

$0.35t = 63$

$t = 180$ people

Step 2 Find the number of people who answered "no opinion."

Let n represent the number of "no opinion" answers. Let d, s, g, and a represent the number of "disagree," "strongly disagree," "strongly agree," and "agree" answers.

$t = n + d + s + g + a$

$180 = n + 63 + 45 + 36 + 27$ *Substitute the numbers shown on the graph.*

$180 = n + 171$ *Simplify the right side of the equation.*

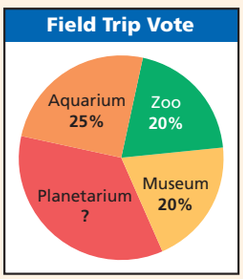
$-171 \quad -171$ *Subtract 171 from both sides.*

$9 = n$

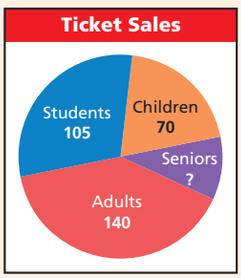
There were 9 people who had no opinion.

Try This

3. The students in a junior high school voted on their choice for a field trip. Sixteen students voted for the natural history museum. How many students voted for the winning choice? **28**



4. At the fall dance recital, 40% of the tickets were sold to adults. What percent of the sales were to seniors? **10%**



Close

Assess

After students have solved the problems, have them make a circle graph for the data in **Example 1** and a bar graph for the data in **Example 2.** Discuss when data should be displayed in a bar graph and when a circle graph is a better display.

Connecting Algebra to Data Analysis **685**

10-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Organize data in tables and graphs.

Choose a table or graph to display data.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Write the equivalent percent.

1. $\frac{4}{16}$ 25%
2. $\frac{12}{60}$ 20%
3. $\frac{35}{280}$ 12.5%

Find each value.

4. 20% of 360 72
5. 75% of 360 270
6. $\frac{1}{15}$ of 360 24

Also available online

Math Humor

Scout Leader: I think we're lost. Someone hand me a compass.

Scout: How is a circle graph going to help us?

COMMON CORE

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Learn It Online
State Resources Online

COMMON CORE

10-1 Organizing and Displaying Data

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Objectives
Organize data in tables and graphs.

Choose a table or graph to display data.

Vocabulary
bar graph
line graph
circle graph

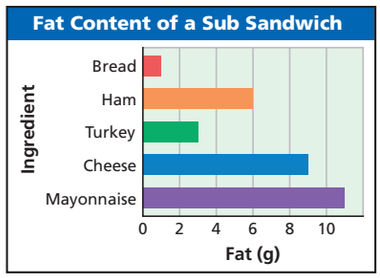
Who uses this?
Nutritionists can display health information about food in bar graphs.

Bar graphs, *line graphs*, and *circle graphs* can be used to present data in a visual way.

A **bar graph** displays data with vertical or horizontal bars. Bar graphs are a good way to display data that can be organized into categories. Using a bar graph, you can quickly compare the categories.

EXAMPLE 1 Reading and Interpreting Bar Graphs

Use the graph to answer each question.



A **Which ingredient contains the most fat?**
mayonnaise — *The bar for mayonnaise is the longest.*

B **How many more grams of fat are in ham than in turkey?**
$6 - 3 = 3$ — *There are 6 grams of fat in ham and 3 grams of fat in turkey.*

C **How many total fat grams are in this sandwich?**
$1 + 6 + 3 + 9 + 11 = 30$ — *Add the number of fat grams for each ingredient.*

D **What percent of the total fat grams in this sandwich are from turkey?**
$\frac{3}{30} = \frac{1}{10} = 10\%$ — *Out of 30 total fat grams, 3 fat grams are from turkey.*

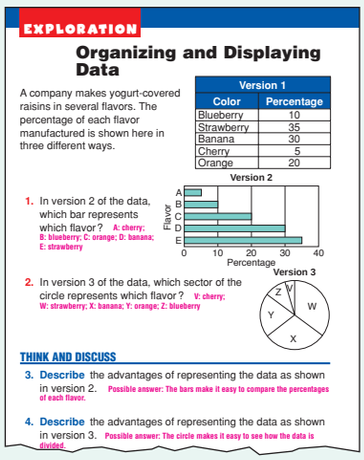
CHECK IT OUT! **Use the graph to answer each question.**

1a. Which ingredient contains the least amount of fat? bread

1b. Which ingredients contain at least 8 grams of fat? cheese and mayonnaise

Victoria Smith/HMH

1 Introduce


EXPLORATION

Organizing and Displaying Data

A company makes yogurt-covered raisins in several flavors. The percentage of each flavor manufactured is shown here in three different ways.

Version 1

Color	Percentage
Blueberry	10
Strawberry	35
Banana	30
Cherry	5
Orange	20

Version 2

1. In version 2 of the data, which bar represents which flavor? A: cherry; B: blueberry; C: orange; D: banana; E: strawberry

Version 3

2. In version 3 of the data, which sector of the circle represents which flavor? V: cherry; W: strawberry; X: banana; Y: orange; Z: blueberry

THINK AND DISCUSS

3. **Describe** the advantages of representing the data as shown in version 2. Possible answer: The bars make it easy to compare the percentages of each flavor.

4. **Describe** the advantages of representing the data as shown in version 3. Possible answer: The circle makes it easy to see how the data is divided.

Motivate

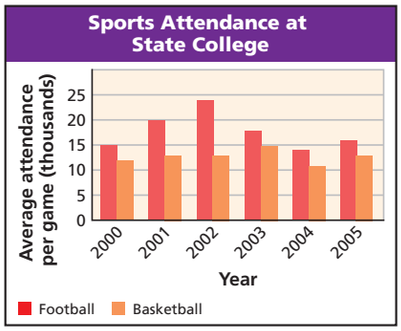
Find examples of data displays such as bar graphs, line graphs, and circle graphs in news magazines or newspapers. Show them to students and discuss some of the many ways that graphs are used in the real world. Have students list careers in which analyzing data is important, such as science, market research, finance, and politics.

Explorations and answers are provided in your online resources.

A double-bar graph can be used to compare two data sets. A double-bar graph has a key to distinguish between the two sets of data.

EXAMPLE 2 Reading and Interpreting Double Bar Graphs

Use the graph to answer each question.



A **In which year did State College have the greatest average attendance for basketball?**

2003

Find the tallest orange bar.

B **On average, how many more people attended a football game than a basketball game in 2001?**

20,000 − 13,000 = 7000

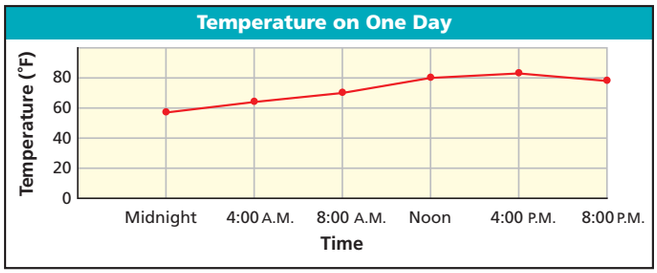
Find the height of each bar for 2001 and subtract.

CHECK IT OUT! **2.** Use the graph to determine which years had the same average basketball attendance. What was the average attendance for those years? **2001, 2002, and 2005; about 13,000**

A **line graph** displays data using line segments. Line graphs are a good way to display data that changes over a period of time.

EXAMPLE 3 Reading and Interpreting Line Graphs

Use the graph to answer each question.



A **At what time was the temperature the warmest?**

4:00 P.M. *Identify the highest point.*

B **During which 4-hour time period did the temperature increase the most?**

From 8:00 A.M. to noon *Look for the segment with the greatest positive slope.*

CHECK IT OUT! **3.** Use the graph to estimate the difference in temperature between 4:00 A.M. and noon. **about 18 °F**

2 Teach

Guided Instruction

Show the importance of labeling the axes. In **Example 2** the interval on the vertical axis is marked in increments of 5, but the axis label shows that each increment represents 5000. In **Example 4,** it may seem obvious that the price is in dollars, but that might not be clear if the graph appeared in an international magazine.

Teaching Tip **Visual Cues** To help find values on bar and line graphs, use a straightedge to line up the bars and points with the axes.

Reaching All Learners *Through Cognitive Strategies*

Have students copy and use this chart to help with reading the word problems. Encourage them to add rows for other actions such as divide, subtract, and multiply.

Cue Words	Action
most, least, -est	Look for extremes.
same	Look for equal values.
more, less/fewer	Make a comparison.
total, altogether	Add.

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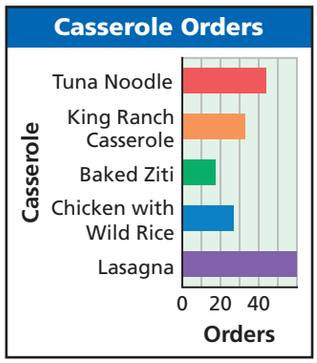
Additional Examples

Example 1

Use the graph to answer each question.



A. Which casserole was ordered the most? lasagna

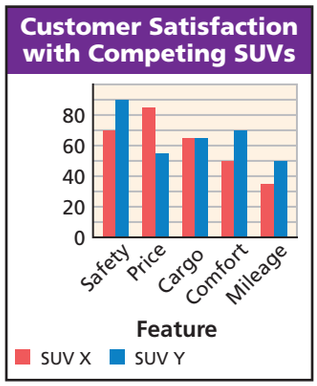
B. About how many total orders were placed? 180

C. About how many more tuna noodle casseroles were ordered than king ranch casseroles? 10

D. About what percent of the total orders were for baked ziti? 10%

Example 2

Use the graph to answer each question.



A. Which feature received the same satisfaction rating for each SUV? cargo

B. Which SUV received a better rating for mileage? SUV Y

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLES 1–2

- Why is the data displayed in these particular graphs?

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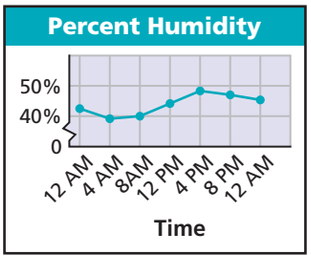
Additional Examples

Example 3

Use the graph to answer each question.

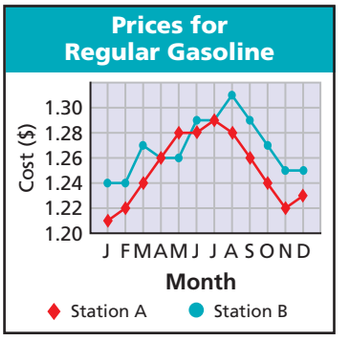


A. At what time was the humidity the lowest? 4 A.M.

B. During which 4-hour time period did the humidity increase the most? 12 to 4 P.M.

Example 4

Use the graph to answer each question.

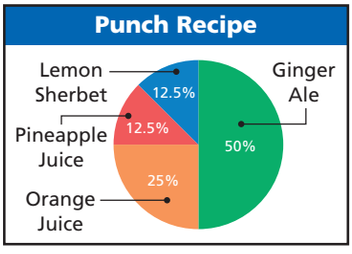


A. In which month did station A charge more than station B? May

B. During which month(s) did the stations charge the same for gasoline? April, July

Example 5

Use the graph to answer the question.



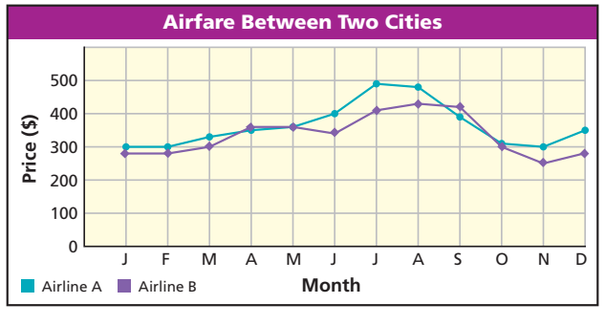
Which ingredients are present in equal amounts? lemon sherbet and pineapple juice

Also available online

A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

EXAMPLE 4 Reading and Interpreting Double-Line Graphs

Use the graph to answer each question.



A **In which month(s) did airline B charge more than airline A?**
April and September *Identify the points when the purple line is higher than the blue line.*

B **During which month(s) did the airlines charge the same airfare?**
May *Look for the point where the data points overlap.*

CHECK IT OUT!

4. Use the graph to describe the general trend of the data.
Prices increased from Jan through Jul or Aug, and then prices decreased through Nov.

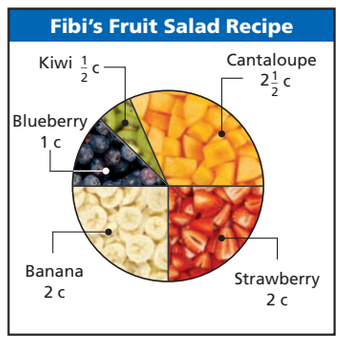
A **circle graph** shows parts of a whole. The entire circle represents 100% of the data and each sector represents a percent of the total. Circle graphs are good for comparing each category of data to the whole set.

EXAMPLE 5 Reading and Interpreting Circle Graphs

Use the graph to answer each question.

A **Which two fruits together make up half of the fruit salad?**
bananas and strawberries
Look for two fruits that together make up half of the circle.

B **Which fruit is used more than any other?**
cantaloupe
Look for the largest sector of the graph.



Reading Math
The sections of a circle graph are called *sectors*.

CHECK IT OUT!

5. Use the graph to determine what percent of the fruit salad is cantaloupe. **31.25%**

Sam Dudgeon/HMH

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLES 3–4

- Why is the data in **Example 3** displayed in a line graph?
- Why do the graphs in **Examples 2** and **4** have a key? Do the graphs in **Examples 1** and **3** need a key?

EXAMPLE 5

- What does the size of each sector indicate?
- When would you use a circle graph instead of a bar graph?

EXAMPLE 6 Choosing and Creating an Appropriate Display

Use the given data to make a graph. Explain why you chose that type of graph.

A

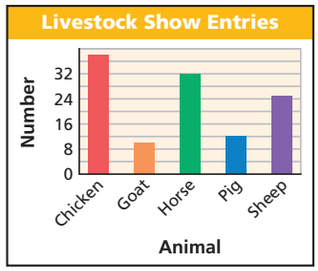
Livestock Show Entries	
Animal	Number
Chicken	38
Goat	10
Horse	32
Pig	12
Sheep	25

A bar graph is appropriate for this data because it will be a good way to compare categories.

Step 1 Determine an appropriate scale and interval. The scale must include all of the data values. The scale is separated into equal parts, called intervals.

Step 2 Use the data to determine the lengths of the bars. Draw bars of equal width. The bars should not touch.

Step 3 Title the graph and label the horizontal and vertical scales.



B

Division of Crops	
Crop	Area (acres)
Corn	70
Fallow	50
Mixed vegetables	10
Soybeans	40
Wheat	30

A circle graph is appropriate for this data because it shows categories as parts of a whole.

Step 1 Calculate the percent of the total represented by each category.

Corn: $\frac{70}{200} = 0.35 = 35\%$

Soybeans: $\frac{40}{200} = 0.2 = 20\%$

Fallow: $\frac{50}{200} = 0.25 = 25\%$

Wheat: $\frac{30}{200} = 0.15 = 15\%$

Mixed vegetables: $\frac{10}{200} = 0.05 = 5\%$

Step 2 Find the angle measure for each sector of the graph. Since there are 360° in a circle, multiply each percent by 360°.

Corn: $0.35 \times 360° = 126°$

Fallow: $0.25 \times 360° = 90°$

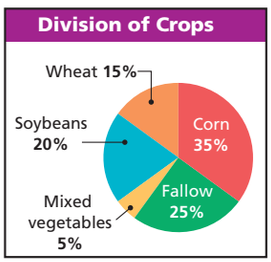
Mixed vegetables: $0.05 \times 360° = 18°$

Soybeans: $0.2 \times 360° = 72°$

Wheat: $0.15 \times 360° = 54°$

Step 3 Use a compass to draw a circle. Mark the center and use a straightedge to draw one radius. Then use a protractor to draw each central angle.

Step 4 Title the graph and label each sector.



COMMON ERROR ALERT

Students may have trouble deciding whether to display data in a bar or circle graph. If the data represents a whole, a circle graph is probably a better choice. If the data displays several categories that do not make up a whole, a bar graph is probably a better choice.

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Additional Examples

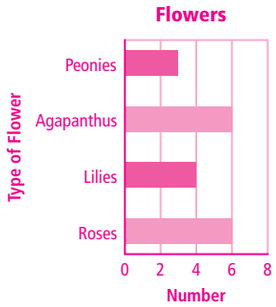
Example 6

Use the given data to make a graph. Explain why you chose that type of graph.

A.

Flower	Number
Roses	6
Lilies	4
Agapanthus	6
Peonies	3

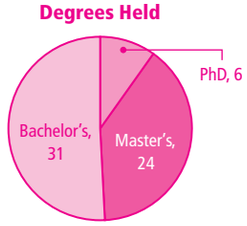
A bar graph is good for displaying categories that do not make up a whole.



B.

Degree Held	Faculty Members
PhD	6
Master's	24
Bachelor's	31

A circle graph is good for displaying categories that make up a whole.



Also available online

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Additional Examples

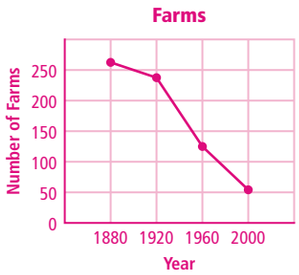
Example 6

Use the given data to make a graph. Explain why you chose that type of graph.

C.

Year	Farms
1880	265
1920	238
1960	124
2000	53

A line graph is good for displaying how something changes over time.



Also available online

Questioning Strategies

EXAMPLE 6

- How do you determine the interval to use on a bar or line graph?
- How do you determine the angle measurements for a circle graph?

Teaching Tip **Inclusion** Axis labels should describe the data and the units of measurement if applicable. The title of a graph should quickly summarize what the graph displays.

Use the given data to make a graph. Explain why you chose that type of graph.

C

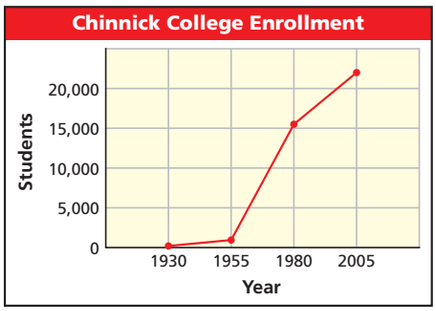
Chinnick College Enrollment	
Year	Students
1930	586
1955	2,361
1980	15,897
2005	21,650

A line graph is appropriate for this data because it will show the change in enrollment over a period of time.

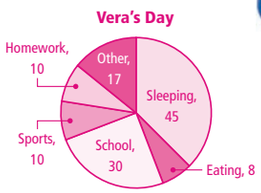
Step 1 Determine the scale and interval for each set of data. Time should be plotted on the horizontal axis because it is independent.

Step 2 Plot a point for each pair of values. Connect the points using line segments.

Step 3 Title the graph and label the horizontal and vertical scales.



6.



A circle graph shows parts of a whole.

CHECK IT OUT! **6. Use the given data to make a graph. Explain why you chose that type of graph.**

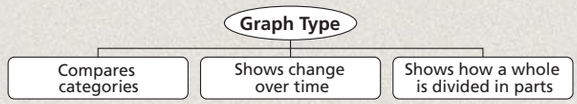
The data below shows how Vera spends her time during a typical 5-day week during the school year.

Vera's Schedule						
Activity	Sleeping	Eating	School	Sports	Homework	Other
Time (h)	45	8	30	10	10	17

THINK AND DISCUSS

1. What are some comparisons you can make by looking at a bar graph?
2. Name some key components of a good line graph.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, tell which kind of graph is described.

Know it! Note



3 Close

Summarize

Review the characteristics of each type of graph.

Bar/Double-Bar Graph: data organized into categories for the purpose of comparing categories

Line/Double-Line Graph: data that changes over a period of time

Circle Graph: data organized into categories for the purpose of comparing each category to the whole set

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. You can compare quantities.
2. The horizontal and vertical scales start at 0, time is placed on the horizontal axis, and even intervals are used between items on the horizontal axis.
3. See Additional Answers.

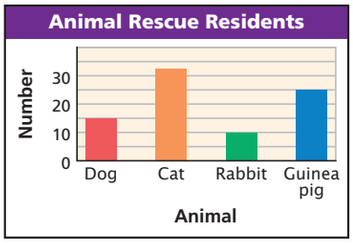
Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Vocabulary Use the vocabulary from this lesson to answer the following questions.

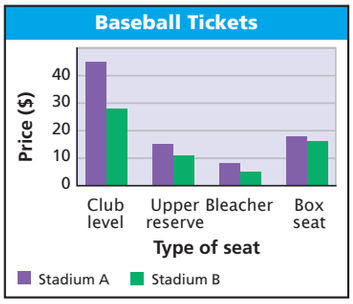
1. In a *circle graph*, what does each sector represent? one part of a whole
2. In a *line graph*, how does the slope of a line segment relate to the rate of change? 2. As the steepness of the line increases, the rate of change increases.

SEE EXAMPLE 1 Use the bar graph for Exercises 3 and 4.



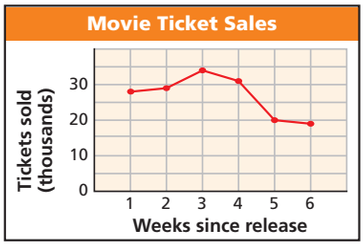
3. Estimate the total number of animals at the shelter. 82 animals
4. There are 3 times as many ___?___ as ___?___ at the animal shelter. cats; rabbits

SEE EXAMPLE 2 Use the double-bar graph for Exercises 5–7.



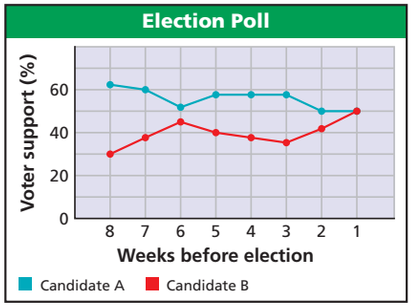
5. About how much more is a club level seat at stadium A than at stadium B? $15
6. Which type of seat is the closest in price at the two stadiums? box seats
7. Describe one relationship between the ticket prices at stadium A and stadium B. Prices at stadium A are greater than prices at stadium B.

SEE EXAMPLE 3 Use the line graph for Exercises 8 and 9.



8. Estimate the number of tickets sold during the week of the greatest sales. 34,000 tickets
9. Which one-week period of time saw the greatest change in sales? between weeks 4 and 5

SEE EXAMPLE 4 Use the double-line graph for Exercises 10–12.



10. When was the support for the two candidates closest? 1 week before the election
11. Estimate the difference in voter support for the two candidates five weeks before the election. 18%
12. Describe the general trend(s) of voter support for the two candidates.

10-1 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–4**
Basic 17–26
Average 17–26
Advanced 17–26

If you finished Examples **1–6**
Basic 17–35, 37–39
Average 17–29, 34–42
Advanced 17–29, 34–42

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 20, 22, 24, 28, 29, 32

Answers

12. Support for candidate B generally increased. Support for candidate A generally decreased.

MATHEMATICAL PRACTICES

Reason abstractly and quantitatively. Exercises 40–42

Construct viable arguments and critique the reasoning of others. Exercises 7, 12, 16, 29, 34, 36

Use appropriate tools strategically. Exercises 29–33

Attend to precision. Exercises 3, 8, 11, 17–18

Learn It Online
State Resources Online

Teaching Tip

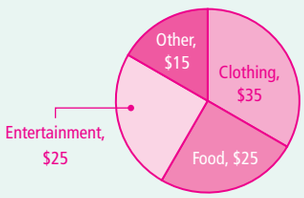
Social Studies Link **Exercises 17 and 18** show populations of Native American tribes. The Cherokee now live mostly in Oklahoma, while the Chippewa, also called Ojibwe or Ojibwa, live primarily in the Great Lakes area. Some Choctaw live in Mississippi and Alabama as well as in Oklahoma. The Navajo Nation is located on the borders of the states of Arizona, New Mexico, and Utah, and the Sioux tribes, also called Lakota, live mostly in the Dakotas.

Teaching Tip

Reading Math Tell students that the word *overall* in **Exercise 19** indicates a sum. Students will need to add the number of customers for lunch and dinner to answer the question. Although Saturday had the most dinner customers, there were relatively few lunch customers.

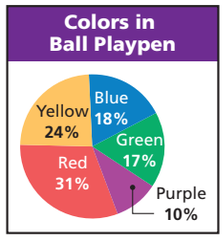
Answers

16. Karim's Budget



SEE EXAMPLE 5

Use the circle graph for Exercises 13–15.



13. Which color is least represented in the ball playpen? **purple**
14. There are 500 balls in the playpen. How many are yellow? **120**
15. Which two colors are approximately equally represented in the ball playpen? **blue and green**

SEE EXAMPLE 6

16. The table shows the breakdown of Karim's monthly budget of $100. Use the given data to make a graph. Explain why you chose that type of graph.

Item/Activity	Spending ($)
Clothing	35
Food	25
Entertainment	25
Other	15

A circle graph compares how much of Karim's total budget is spent in each category.

PRACTICE AND PROBLEM SOLVING

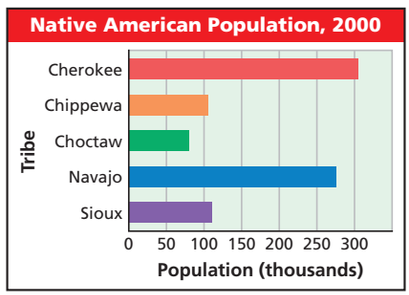
Independent Practice

For Exercises	See Example
17–18	1
19–21	2
22–23	3
24–26	4
27–28	5
29	6

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

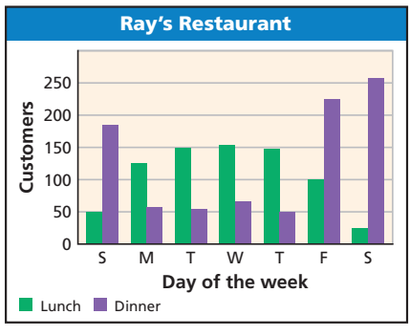
Use the bar graph for Exercises 17 and 18.



Source: U.S. Census Bureau

17. Estimate the difference in population between the tribes with the largest and the smallest population. **225,000**
18. Approximately what percent of the total population shown in the table is Cherokee? **about 35%**

Use the double bar graph for Exercises 19–21.



19. On what day did Ray do the most overall business? **Fri**
20. On what day did Ray have the busiest lunch? **Wed**
21. On Sunday, about how many times as great was the number of dinner customers as the number of lunch customers? **3.5 times**

Use the line graph for Exercises 22 and 23.

22. Between which two games did Marlon's score increase the most? **games 1 and 2**
23. Between which three games did Marlon's score increase by about the same amount? **games 3, 4, and 5**

PRACTICE A

PRACTICE C

PRACTICE B

Look at the double bar graph.

1. Which was the first year that the Barnes rented more DVDs than VHS tapes?
   2005
2. About how many videos did the Barnes family rent in all in 2003?
   53

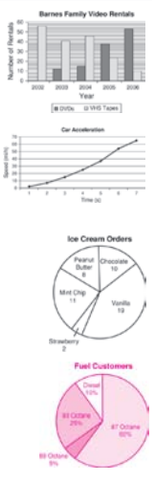
Look at the line graph.

3. During which time interval did the car's speed increase the least?
   1 to 2 s
4. Describe how the speed changed over time.
   It always increased, but from 0–6 s, the speed increased by growing amounts.

Look at the circle graph.

5. There were 5 times the number of orders for **chocolate** as there were for strawberry.
6. What percent of the orders for ice cream were for mint chip or vanilla? **60%**
7. The table shows the number of customers who pumped 4 types of fuel at a gas station in a given time period. Use the given data to make a graph. Explain why you chose that type of graph.

87 Octane	89 Octane	93 Octane	Diesel
12	1	5	2



READING STRATEGIES

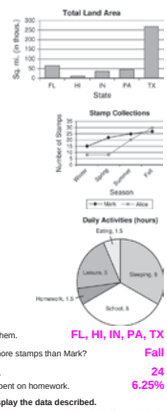
Depending on the data, one kind of graph is usually more appropriate than another.

Bar Graph
- Used for data that can be organized into *categories*.
- Makes it easy to make *comparisons*.

A **double bar graph** can be used to compare two sets of data.

Line Graph
- Used for displaying data that changes over a period of *time*.
- Makes it easy to see *change*.
- A **double line graph** is used to compare two related sets of data that change over time.

Circle Graph
- Used for data that represents *part of a whole*.
- Makes it easy to compare each category to the whole set.
- The whole circle represents 100% and each sector represents a *percent of the total*.



Complete the following.

1. There are five categories in the bar graph. List them. **FL, HI, IN, PA, TX**
2. Look at the line graph. When does Alice have more stamps than Mark? **Fall**
3. Add the hours in each sector of the circle graph. **24**
   Divide 1.5 by 24 to find the percent of the day spent on homework. **6.25%**

Name the type of graph that would be best to display the data described.

4. the cost of two brands of cereal over time — **double line graph**
5. the percent of a person's budget spent on various things — **circle graph**
6. the life expectancy of several species of reptile — **bar graph**
7. the change in school enrollment from 2000 to 2005 — **line graph**

RETEACH

If data:	Then use:
is organized into categories	bar graph/double bar graph
changes over a period of time	line graph/double line graph
compares categories to whole set	circle graph

Use the data at right to make a graph. Explain why you chose that type of graph.

Because the data compares categories (the ingredients) to a whole set (the recipe), a circle graph is best.

Step 1: Total the number of cups.
$8 + 1 + 1 + 10 = 20$

Step 2: Calculate the percent of each ingredient.
- bubble soap: $\frac{8}{20} = 40\%$
- glycerin: $\frac{1}{20} = 5\%$
- dishwashing liquid: $\frac{1}{20} = 5\%$
- water: $\frac{10}{20} = 50\%$

Recipe for Bubbles	
Ingredient	Cups
bubble soap	8
dishwashing liquid	1
glycerin	1
water	10

Step 3: Find the angle measure for each sector of the graph.
- bubble soap: 40% (360°) = 144°
- glycerin: 5% (360°) = 18°
- dishwashing liquid: 5% (360°) = 18°
- water: 50% (360°) = 180°

Use a compass and protractor to draw the graph.

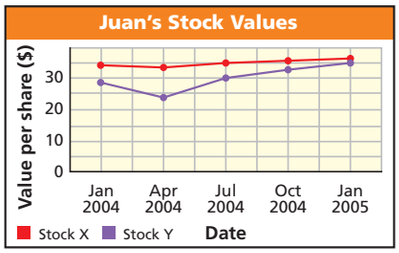
Write *bar*, *double-bar*, *line*, *double-line*, or *circle* to indicate the type of graph that would best display the data described.

1. math scores of one student over the school year — **line**
2. attendance at an exercise class by age group, as it relates to total attendance — **circle**
3. number of animals seen at a farm — **bar**
4. A store owner uses an entire wall to display toys as shown in the table. Use the data to make a graph. Then explain why you chose that type of graph.

Toys for Sale	
Toy	Shelves
games	10
puzzles	5
dolls	3
trains	2

A circle graph is appropriate because it shows categories as part of a whole.

Use the double-line graph for Exercises 24–26.



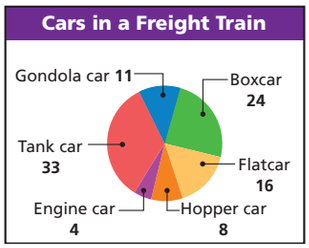
24. What was the average value per share of Juan's two stocks in July 2004? **$32.50**

25. Which stock's value changed the most over any time period?

25. Stock Y changed the most between Apr. and Jul. of 2004.

26. Describe the trend of the values of both stocks. **Both generally increased.**

Use the circle graph for Exercises 27 and 28.



27. About what percent of the total number of cars are hopper cars? **$8\frac{1}{3}\%$**

28. About what percent of the total number of cars are gondola or tank cars? **about 46%**

29. The table shows the weight of twin babies at various times from birth to four weeks old. Use the given data to make a graph. Explain why you chose that type of graph.

Age (days)	Boy's Weight (lb)	Girl's Weight (lb)
1	5.3	5.7
3	5.0	5.2
7	5.5	5.9
14	6.2	6.8
28	7.9	7.5

A double-line graph shows the change over time for 2 things. It also allows you to compare the 2 things.

Write *bar, double-bar, line, double-line,* or *circle* to indicate the type of graph that would best display the data described.

30. attendance at a carnival each year over a ten-year period **line**

31. attendance at two different carnivals each year over a ten-year period **double line**

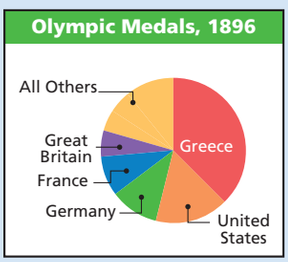
32. attendance at five different carnivals during the same year **bar**

33. attendance at a carnival by age group as it relates to total attendance **circle**

34. Critical Thinking Give an example of real-world data that would best be displayed by each type of graph: line graph, circle graph, double-bar graph.

MULTI-STEP TEST PREP

35. The first modern Olympic Games took place in 1896 in Athens, Greece. The circle graph shows the total number of medals won by several countries at the Olympic Games of 1896.



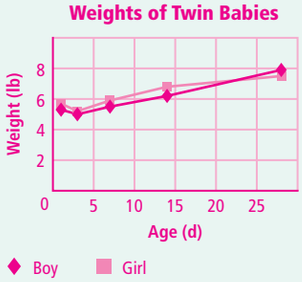
a. Which country won the most medals? Estimate the percent of the medals won by this country. **Greece; about 40%**

b. Which country won the second most medals? Estimate the percent of the medals won by this country. **United States; about 15%**

MULTI-STEP TEST PREP **Exercise 35** involves reading a circle graph. This exercise prepares students for the Multi-Step Test Prep.

Answers

29.



A double-line graph shows the change over time for 2 things. It also allows you to compare the 2 things.

34. Possible answer: line graph: your height over time; circle graph: your CD collection divided into categories; double-bar graph: number of hours you spend on homework or sports each day of the week

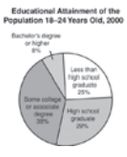
PROBLEM SOLVING

The table below gives the top five ringtone genres reported during a survey of cell phone subscribers in August, 2005.

Ringtone genre	Males (millions)	Females (millions)
Hip Hop or Rap	2.232	2.472
Rock or Alternative	2.219	2.082
Pop	1.044	2.069
TV/Movie/Game Themes	1.161	1.371
Classic Rock	0.949	0.918

1. Use the data to make a graph below. Explain why you chose that type of graph.
   A double-bar graph compares the number of males and females that prefer each of the five categories.
2. From the graph (*not* the table), explain how you can tell that the Pop genre is nearly twice as popular with females as males.
   In the category of Pop, the bar for females is twice as tall as the bar for males.

In 2000, there were approximately 27 million people 18–24 years old in the United States. The circle graph below shows the highest level of education for that population. Use the graph to select the best answers for questions 3–5.



3. Which level of education was least represented?
   A Less than high school graduate
   B Some college or associate degree
   C High school graduate
   D Bachelor's degree or higher
4. How many 18–24 year-olds had attained a bachelor's degree or higher?
   F 2.16 million H 10.26 million
   G 8 million J 12.42 million
5. How many 18–24 year-olds never attended college?
   A 6.75 million C 12.42 million
   B 7.83 million **D** 14.58 million

CHALLENGE

A **mosaic display** is related to both the bar graph and the circle graph. In a mosaic display, each bar is the same height, but different widths. The widths of the bars tell you the percents of the total that fall into each category. Each bar may also be divided into sections that tell you the percents of that category that fall into other categories.

In this activity, you will read and create mosaic displays about the 1912 voyage of the *Titanic*.

The mosaic display above shows survival on the *Titanic* by gender. (This data includes passengers and crew.)

1. Compare the width of the bar for males to the width of the bar for females. Estimate what percent of the passengers on the Titanic were female. **about 20%**
2. Compare the height of the section for females who survived to the height of the section for females who died. Estimate what percent of the female passengers survived. **about 75%**
3. The total number of male survivors (367) was about equal to the number of female survivors (344). How can you tell this from the mosaic display?
   The area of the section for female survivors is about the same as the area for the section of male survivors.
4. The total number of females who died was 126. Estimate the number of males that died, and explain how you got your answer. **about 1260;**
   The area of the section for males who died is about 10 times the size of the area of the section for females who died.

The table at right shows survival on the *Titanic* by passenger class. (This data does *not* include crew.) Follow these steps to make a mosaic display.

	1st	2nd	3rd
Survived	203	118	178
Died	122	167	528

5. Calculate the percentages of all passengers who survived and died. Use the percentages to make two bars of appropriate widths.
6. Calculate the percentage of survivors that were in each class. Use the percentages to divide the bar for survivors into sections of appropriate heights.
7. Calculate the percentage of non-survivors that were in each class. Use the percentages to divide the other bar into sections.

TEST PREP DOCTOR In **Exercise 37,** the words *contribution* and *each* show that the focus is on the *parts* of the team's *total* points, so a circle graph is most appropriate.

Students who chose **J** in **Exercise 38** found 75% of 4 but did not add that amount to 4.

Journal

Have students describe how to make a circle graph.

ALTERNATIVE ASSESSMENT

Have students collect real-world data and display it in an appropriate graph.

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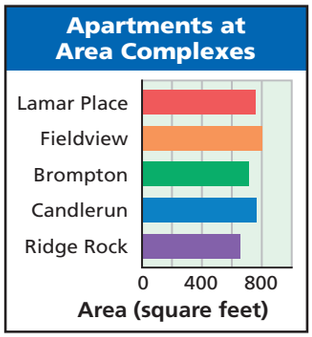
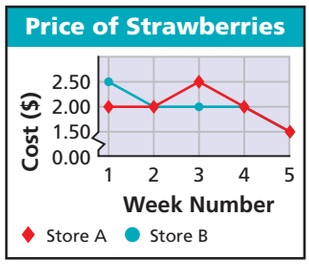
10-1 Lesson Quiz

1. Which two apartments are about the same size?



2. In which week(s) did store B charge more than store A?



3. The table shows how many orders were placed for each type of muffin at a bakery in one week. Use the data to make a graph. Explain why you chose that type of graph.

Type	Number
Banana nut	69
Blueberry	20
Cinnamon chip	25
Lemon poppy seed	11

For answers, see Additional Answers.

Also available online

36. Write About It Explain how you could use a line graph to make predictions.
If the graph is generally increasing, you can predict that the value in the future will increase.

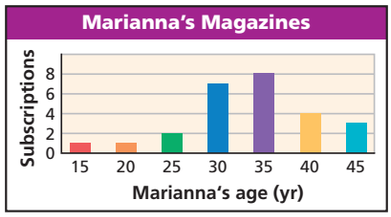
TEST PREP

37. Which type of graph would best display the contribution of each high school basketball player to the team, in terms of points scored?

Ⓐ Bar graph Ⓑ Line graph Ⓒ Double-line graph Ⓓ Circle graph

38. At what age did Marianna have 75% more magazine subscriptions than she did at age 40?

Ⓕ 25
Ⓖ 30
Ⓗ 35
Ⓙ 45

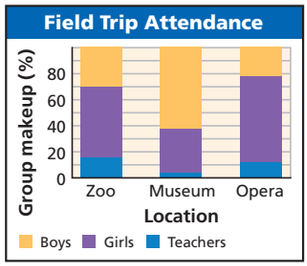


39. Short Response The table shows the number of students in each algebra class. Make a graph to display the data. Explain why you chose that type of graph.

Teacher	Students
Mr. Abrams	34
Ms. Belle	29
Mr. Marvin	25
Ms. Swanson	27

CHALLENGE AND EXTEND

Students and teachers at Lauren's school went on one of three field trips.



40. On which trip were there more boys than girls? museum

41. A total of 60 people went to the museum. Estimate the number of girls who went to the museum. 19 girls

42. Explain why it is not possible to determine whether fewer teachers went to the museum than to the zoo or the opera.
The graph shows only the percentages of each group made up of teachers, boys, and girls. It does not tell how many people went on each trip.

Answers

39.

A bar graph allows you to compare categories.

COMMON CORE

10-2 Frequency and Histograms

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Objectives
Create stem-and-leaf plots.

Create frequency tables and histograms.

Vocabulary
stem-and-leaf plot
frequency
frequency table
histogram
cumulative frequency

Why learn this?
Stem-and-leaf plots can be used to organize data, like the number of students in elective classes. (See Example 1.)

A **stem-and-leaf plot** arranges data by dividing each data value into two parts. This allows you to see each data value.

The digits other than the last digit of each value are called a stem. → 2|3 ← The last digit of a value is called a leaf.

Key: 2|3 means 23 ← The key tells you how to read each value.

EXAMPLE 1 Making a Stem-and-Leaf Plot

A The numbers of students in each of the elective classes at a school are given below. Use the data to make a stem-and-leaf plot.

24, 14, 12, 25, 32, 18, 23, 24, 9, 18, 34, 28, 24, 27

Number of Students in Elective Classes

Stem	Leaves
0	9
1	2 4 8 8
2	3 4 4 4 5 7 8
3	2 4

Key: 2|3 means 23

The tens digits are the stems.
The ones digits are the leaves. List the leaves from least to greatest within each row.
Title the graph and add a key.

Writing Math
Stems are always consecutive numbers. In Example 1B, neither player has scores that start with 15, so there are no leaves in that row.

B Marty's and Bill's scores for ten games of bowling are given below. Use the data to make a back-to-back stem-and-leaf plot.

Marty: 137, 149, 167, 134, 121, 127, 143, 123, 168, 162

Bill: 129, 138, 141, 124, 139, 160, 149, 145, 128, 130

Bowling Scores

Marty		Bill
7 3 1	12	4 8 9
7 4	13	0 8 9
9 3	14	1 5 9
	15	
8 7 2	16	0

Key: |14|1 means 141
3|14| means 143

The first two digits are the stems.
The ones digits are the leaves.
Put Marty's scores on the left side and Bill's scores on the right.
Title the graph and add a key.
The graph shows that three of Marty's scores were higher than Bill's highest score.

CHECK IT OUT!

1. The temperatures in degrees Celsius for two weeks are given below. Use the data to make a stem-and-leaf plot.

7, 32, 34, 31, 26, 27, 23, 19, 22, 29, 30, 36, 35, 31

1. Temperature (°C)

Stem	Leaves
0	7
1	9
2	2 3 6 7 9
3	0 1 1 2 4 5 6

Key: 1|9 means 19

Tom Stewart/CORBIS

10-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Create stem-and-leaf plots.

Create frequency tables and histograms.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Identify the least and greatest value in each data set.

1. 34, 62, 45, 35, 75, 23, 35, 65, 23 — 23, 75
2. 1.6, 3.4, 2.6, 4.8, 1.3, 3.5, 4.0 — 1.3, 4.8

Order the data from least to greatest.

3. 2.4, 5.1, 3.7, 2.1, 3.6, 4.0, 2.9 — 2.1, 2.4, 2.9, 3.6, 3.7, 4.0, 5.1
4. 5, 5, 6, 8, 7, 4, 6, 5, 9, 3, 6, 6, 9 — 3, 4, 5, 5, 5, 6, 6, 6, 6, 7, 8, 9, 9

Also available online

Math Humor

Q: What did the investigator say when he collected more data?

A: The stem-and-leaf plot thickens!

COMMON CORE

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Frequency and Histograms

A teacher is analyzing the scores from the last exam he gave. The scores are shown here.

65, 73, 99, 94, 68, 52, 69, 72, 97, 88, 72, 88, 84, 95, 97, 97, 91, 85, 77, 74

1. How many students took the exam? 20
2. Order the scores from least to greatest. 52, 65, 68, 69, 72, 72, 73, 74, 77, 84, 85, 88, 88, 91, 94, 95, 97, 97, 97, 99
3. Complete the table by counting the number of scores that lie in each interval.

Scores	Number of Scores in This Interval
90–99	7
80–89	4
70–79	5
60–69	3
50–59	1

4. In which interval do the greatest number of scores lie? 90–99
5. Add the values in the right-hand column of the table. What is the sum? Why does this make sense? 20; Since 20 students took the exam, the total shows that all the scores are accounted for.

THINK AND DISCUSS

6. **Describe** one conclusion that the teacher might make from looking at the table. Possible answer: In general, students did well on the exam.

Motivate

Give students a set of two-digit numbers. Ask students to group the numbers by tens and then to order each group from least to greatest. Explain that these are the first steps in making a stem-and-leaf plot. Show students a histogram of the same data set. Ask students how they could use their groups of numbers to make a histogram. Line up the numbers in each group vertically.

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

A. The numbers of defective widgets in batches of 1000 are given below. Use the data to make a stem-and-leaf plot.
14, 12, 8, 9, 13, 20, 15, 9, 21, 8, 13, 19

Defective Widgets per Batch

Stem	Leaves
0	8 8 9 9
1	2 3 3 4 5 9
2	0 1

Key: 1|2 means 12

B. The season's scores for the football teams going to the state championship are given below. Use the data to make a back-to-back stem-and-leaf plot.
Team A: 65, 42, 56, 49, 58, 42, 61, 55, 45, 72
Team B: 57, 60, 48, 49, 52, 61, 58, 37, 63, 48

Football State Championship Scores

Team A		Team B
	3	7
9 5 2 2	4	8 8 9
8 6 5	5	2 7 8
5 1	6	0 1 3
2	7	

Key: \|4\|8 means 48
2\|4\| means 42

Example 2

The numbers of students enrolled in Western Civilization classes at a university are given below. Use the data to make a frequency table with intervals.
12, 22, 18, 9, 25, 31, 28, 19, 22, 27, 32, 14

Enrollment in Western Civilization Classes	
Number Enrolled	Frequency
1–10	1
11–20	4
21–30	5
31–40	2

Also available online

The **frequency** of a data value is the number of times it occurs. A **frequency table** shows the frequency of each data value. If the data is divided into intervals, the table shows the frequency of each interval.

EXAMPLE 2 Making a Frequency Table

The final scores for each golfer in a tournament are given below. Use the data to make a frequency table with intervals.

77, 71, 70, 82, 75, 76, 72, 70, 77, 74, 71, 75, 68, 72, 75, 74

Step 1 Identify the least and greatest values.

The least value is 68. The greatest value is 82.

Step 2 Divide the data into equal intervals.

For this data set, use an interval of 3.

Step 3 List the intervals in the first column of the table. Count the number of data values in each interval and list the count in the last column. Give the table a title.

Golf Tournament Scores	
Scores	Frequency
68–70	3
71–73	4
74–76	6
77–79	2
80–82	1

2.

Days	Frequency
4–6	5
7–9	4
10–12	4
13–15	2

CHECK IT OUT!

2. The numbers of days of Maria's last 15 vacations are listed below. Use the data to make a frequency table with intervals.
4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

A **histogram** is a bar graph used to display the frequency of data divided into equal intervals. The bars must be of equal width and should touch, but not overlap.

EXAMPLE 3 Making a Histogram

Use the frequency table in Example 2 to make a histogram.

Step 1 Use the scale and interval from the frequency table.

Step 2 Draw a bar for the number of scores in each interval.

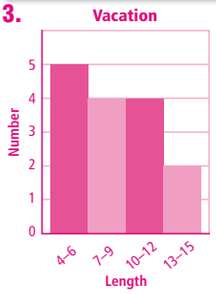
All bars should be the same width. The bars should touch, but not overlap.

Step 3 Title the graph and label the horizontal and vertical scales.

Helpful Hint

The intervals in a histogram must be of equal size.

3.



CHECK IT OUT!

3. Make a histogram for the number of days of Maria's last 15 vacations.
4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

2 Teach

Guided Instruction

Tell students that whether they are creating stem-and-leaf plots or frequency tables, it is helpful to first arrange the data from least to greatest. In **Example 2,** point out that frequency tables should have between four and seven intervals and that each interval should have at least two pieces of data.

Teaching Tip **Visual** A stem-and-leaf plot, rotated 90° counterclockwise, resembles a histogram.

Reaching All Learners

Through Kinesthetic Experience

Allow each student to grab a handful of integer chips. These can be found in the Manipulatives Kit (MK). Each student should count the number of integer chips he or she holds and write that number on a sticky note. Write intervals of 5 across the board and have students place their number above the correct interval. Numbers within the same interval should be lined up vertically. The result resembles a histogram.

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLES 1–3

- How do you decide what the stems should be for a stem-and-leaf plot?
- How can a stem-and-leaf plot help you make a frequency table?

Cumulative frequency shows the frequency of all data values less than or equal to a given value. You could just count the number of values, but if the data set has many values, you might lose track. Recording the data in a cumulative frequency table can help you keep track of the data values as you count.

EXAMPLE 4 Making a Cumulative Frequency Table

The heights in inches of the players on a school basketball team are given below.

72, 68, 71, 70, 73, 69, 79, 76, 72, 75, 72, 74, 68, 70, 69, 75, 72, 71, 73, 76

a. Use the data to make a cumulative frequency table.

Step 1 Choose intervals for the first column of the table.

Step 2 Record the frequency of values in each interval for the second column.

Step 3 Add the frequency of each interval to the frequencies of all the intervals before it. Put that number in the third column of the table.

Step 4 Title the table.

Basketball Players' Heights

Height (in.)	Frequency	Cumulative Frequency
68–70	6	6
71–73	8	14
74–76	5	19
77–79	1	20

b. How many players have heights under 74 in?

All heights under 74 in. are displayed in the first two rows of the table, so look at the cumulative frequency shown in the second row.

There are 14 players with heights under 74 in.

CHECK IT OUT!

4. The numbers of vowels in each sentence of a short essay are listed below.

33, 36, 39, 37, 34, 35, 43, 35, 28, 32, 36, 35, 29, 40, 33, 41, 37

a. Use the data to make a cumulative frequency table.

b. How many sentences contain 35 vowels or fewer? 9

MATHEMATICAL PRACTICES

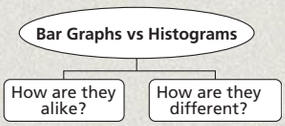
THINK AND DISCUSS

1. In a stem-and-leaf plot, the number of _____?_____ is always the same as the number of data values. (*stems* or *leaves*)

2. Explain how to make a histogram from a stem-and-leaf plot.

3. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note



3 Close

Summarize

Remind students that stem-and-leaf plots, frequency tables, and histograms all show how often a data value, or range of data values, occurs. A benefit of a stem-and-leaf plot is that it shows what the individual data values are. A histogram gives a quick visual representation of where most of the data lie, but it does not reveal the individual data values.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

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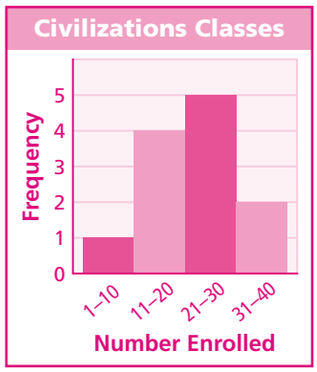
Additional Examples

Example 3

Use the frequency table in **Additional Example 2** to make a histogram.



Example 4

The weights (in ounces) of packages of pork chops are given below.

19, 20, 26, 18, 25, 29, 18, 18, 22, 24, 27, 26, 24, 21, 29, 19

A. Use the data to make a cumulative frequency table.

Pork Chops

Weight (oz)	Frequency	Cumulative Frequency
18–20	6	6
21–23	2	8
24–26	5	13
27–29	3	16

B. How many packages weigh less than 24 ounces? 8

Also available online

Answer to *Check It Out!*

4a. See Additional Answers.

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- What does the last number in the cumulative frequency column represent? Explain.

Answers to *Think and Discuss*

1. leaves
2. The stems are the intervals, and the number of leaves for each stem gives the height of the bar.
3. See Additional Answers.

10-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 7–9
Average 7–9, 21
Advanced 7–9, 21

If you finished Examples **1–4**
Basic 7–16, 18–20
Average 7–11, 13–20
Advanced 7–11, 13, 15–21

Homework Quick Check
Quickly check key concepts.
Exercises: 7, 8, 9, 10, 11, 16

Answers

2.

Stem	Leaves
1	8 9
2	0 1 1 1 1 2 2 2 3 4

Key: 1|8 means 18

3.

Austin		New York
9 9 9	1	
4 3 2 1	2	
6 5 3 2 0	3	1 3 3 3 6 6 7 9
	4	1 1 2 2

Key: \|3\|1 means 3.1
9\|1\| means 1.9

4–6a, 7. See Additional Answers.

MATHEMATICAL PRACTICES

Reason abstractly and quantitatively.
Exercises 6, 11, 15

Construct viable arguments and critique the reasoning of others.
Exercises 13–14, 17

Look for and make use of structure.
Exercise 21

Learn It Online
State Resources Online

GUIDED PRACTICE

1. **Vocabulary** A(n) _____?_____ is a data display that shows individual data values. (*stem-and-leaf plot* or *histogram*) **stem-and-leaf plot**

SEE EXAMPLE 1

2. **Sports** The ages of professional basketball players at the time the players were recruited are given. Use the data to make a stem-and-leaf plot.

Ages When Recruited											
21	23	21	18	22	19	24	22	21	22	20	21

3. **Weather** The average monthly rainfall for two cities (in inches) is given below. Use the data to make a back-to-back stem-and-leaf plot.

Average Monthly Rainfall (in.)												
Austin, TX	1.9	2.4	1.9	3.0	3.6	3.3	1.9	2.1	3.2	3.5	2.2	2.3
New York, NY	3.3	3.1	3.9	3.7	4.2	3.3	4.1	4.1	3.6	3.3	4.2	3.6

SEE EXAMPLE 2

4. **Sports** The finishing times of runners in a 5K race, to the nearest minute, are given. Use the data to make a frequency table with intervals.

Finishing Times in 5K Race (to the nearest minute)								
19	25	23	29	32	30	21	22	24
19	28	26	31	34	30	28	25	24

SEE EXAMPLE 3

5. **Biology** The breathing intervals of gray whales are given. Use the frequency table to make a histogram for the data.

Breathing Intervals (min)	
Interval	Frequency
5–7	4
8–10	7
11–13	7
14–16	8

SEE EXAMPLE 4

6. The scores made by a group of eleventh-grade students on the mathematics portion of the SAT are given.

Scores on Mathematics Portion of SAT							
520	560	720	690	540	630	790	540
600	580	710	500	540	660	630	

   a. Use the data to make a cumulative frequency table.
   b. How many students scored 650 or higher on the mathematics portion of the SAT? **5**

PRACTICE AND PROBLEM SOLVING

7. The numbers of people who visited a park each day over two weeks during different seasons are given below. Use the data to make a back-to-back stem-and-leaf plot.

Visitors to a Park													
Summer	25	25	26	27	27	57	59	22	23	29	22	23	54	53
Winter	11	12	13	9	30	27	4	19	14	19	21	33	35	9

Independent Practice

For Exercises	See Example
7–8	1
9	2
10	3
11	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

8. Weather The daily high temperatures in degrees Fahrenheit in a town during one month are given. Use the data to make a stem-and-leaf plot.

Daily High Temperatures (°F)										
68	72	79	77	70	72	75	71	64	64
68	62	70	71	78	83	83	87	91	89
87	75	73	70	69	69	62	58	71	76

9. The overall GPAs of several high school seniors are given. Use the data to make a frequency table with intervals.

Overall GPAs								
3.6	2.9	3.1	3.0	2.5	2.6	3.8	2.9	
2.2	2.9	3.1	3.3	3.6	3.0	2.3	2.8	2.9

10. Chemistry The atomic masses of the nonmetal elements are given in the table. Use the frequency table to make a histogram for the data.

Atomic Masses of Nonmetal Elements					
Interval	0–49.9	50–99.9	100–149.9	150–199.9	200–249.9
Frequency	11	3	2	0	2

Solar cars usually weigh between 330 and 880 pounds. A conventional car weighs over 4000 pounds.

11. The numbers of pretzels found in several samples of snack mix are given in the table.

Numbers of Pretzels					
42	39	39	38	40	
41	44	42	38	44	
47	36	40	40	43	38

a. Use the data to make a cumulative frequency table.

b. How many samples of snack mix had fewer than 42 pretzels? **10**

12. Automobiles The table shows gas mileage for the most economical cars in July 2004, including three hybrids.

Gas Mileage of Economical Cars									
Mileage in City (mi/gal)	32	60	48	38	36	60	35	38	32
Mileage on Highway (mi/gal)	38	51	47	46	47	66	43	46	40

Make a back-to-back stem-and-leaf plot for the data.

13. Damien's math test scores are given in the table:

Damien's Math Test Scores		
75	84	68
72	59	88
72	77	81
84	60	70

a. Make a stem-and-leaf plot of Damien's test scores.

b. Make a histogram of the test scores using intervals of 5.

c. Make a histogram of the test scores using intervals of 10.

d. Make a histogram of the test scores using intervals of 20.

e. How does the size of the interval affect the appearance of the histogram?

f. Write About It Which histogram makes Damien's grades look highest? Explain.

14. ERROR ANALYSIS Two students made stem-and-leaf plots for the following data: 530, 545, 550, 555, 570. Which is incorrect? Explain the error. **A; there is no stem for 56.**

A

Stem	Leaves
53	0
54	5
55	0 5
57	0

Key: 52|5 means 525

B

Stem	Leaves
53	0
54	5
55	0 5
56	
57	0

Key: 52|5 means 525

© Reuters/CORBIS

COMMON ERROR ALERT

Students might choose whole number intervals for **Exercise 9,** which are too large. Explain that intervals can be as small or as large as necessary.

Teaching Tip **Inclusion** Remind students that each value must be in only one interval. For instance, they should use 10–19 and 20–29 instead of 10–20 and 20–30.

Answers

8.

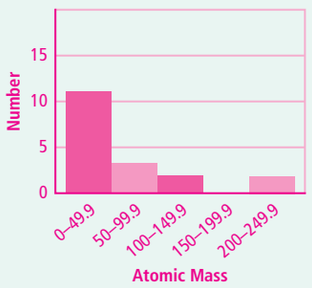
Stem	Leaves
5	8
6	2 2 4 4 8 8 9 9
7	0 0 0 1 1 1 2 2 3 5 5 6 7 8 9
8	3 3 7 7 9
9	1

Key: 5|8 means 58

9.

Interval	Frequency
2.0–2.4	2
2.5–2.9	7
3.0–3.4	5
3.5–3.9	3

10.



11a.

Interval	Frequency	Cumulative Frequency
36–38	4	4
39–41	6	10
42–44	5	15
45–47	1	16

12.

City Driving		Highway Driving
8 8 6 5 2 2	3	8
8	4	0 3 6 6 7 7
	5	1
0 0	6	6

Key: |3|8 means 38

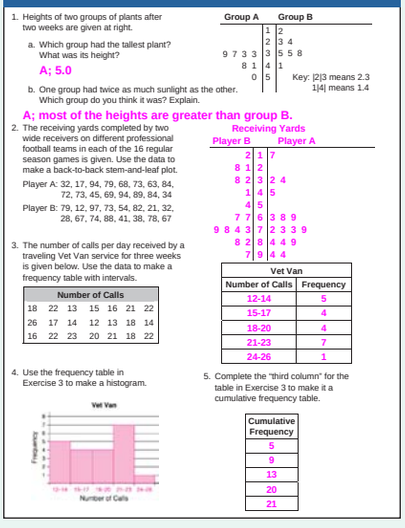
8|4| means 48

13. See Additional Answers.

PRACTICE A

PRACTICE C

PRACTICE B



MULTI-STEP TEST PREP **Exercise 15** involves organizing data for display in a graph. This exercise prepares students for the Multi-Step Test Prep.

TEST PREP DOCTOR If students answered **Exercise 18** incorrectly, draw their attention to the key. Remind them that in this plot, the stem is the ones place and the leaf is the tenths place of a data value. The only number that is expressed in tenths is 4.7, choice **B.**

Students who chose **F** for **Exercise 19** probably referred to the cumulative frequency column instead of the frequency column.

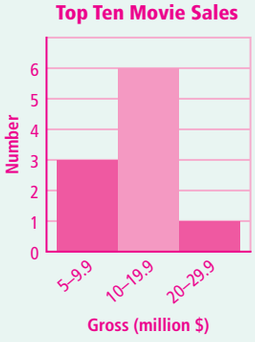
Answers

15a.

Interval	Frequency
160–169.9	2
170–179.9	4
180–189.9	3
190–199.9	1
200–209.9	2
210–219.9	1

b.

16.



MULTI-STEP TEST PREP

15. The 2004 Olympic results for women's weightlifting in the 48 kg weight class are 210, 205, 200, 190, 187.5, 182.5, 180, 177.5, 175, 172.5, 170, 167.5, and 165, measured in kilograms. Medals are awarded to the athletes who can lift the most weight.

a. Create a frequency table beginning at 160 and using intervals of 10 kg.

b. Create a histogram of the data.

c. Tara Cunningham from the United States lifted 172.5 kg. Did she win a medal? How do you know? **No; she did not lift 1 of the 3 greatest weights.**

16. Entertainment The top ten movies in United States theaters for the weekend of June 25–27, 2004, grossed the following amounts (in millions of dollars). Create a histogram for the data. Make the first interval 5–9.9.

Ticket Sales (million $)				
23.9	19.7	18.8	13.5	13.1
11.2	10.2	7.5	6.1	5.1

17. Critical Thinking Margo's homework assignment is to make a data display of some data she finds in a newspaper. She found a frequency table with the given intervals.

Explain why Margo must be careful when drawing the bars of the histogram.

17. The intervals are not the same size, so the histogram may not accurately represent ages under 18 or over 55.

Age
Under 18
18–30
31–54
55 and older

TEST PREP

18. What data value occurs most often in the stem-and-leaf plot?

Ⓐ 7 Ⓒ 47

Ⓑ 4.7 Ⓓ 777

Stem	Leaves
3	2 3 4 4 7 9
4	0 1 5 7 7 7 8
5	1 2 2 3

Key: 3|2 means 3.2

19. The table shows the results of a survey about time spent on the Internet each month. Which statement is NOT supported by the data in the table?

Time Spent on the Internet per Month		
Time (h)	Frequency	Cumulative Frequency
0–4	4	4
5–9	6	10
10–14	3	13
15–19	16	29
20–24	12	41
25–29	7	48
30–34	2	50

Ⓕ The interval of 30 to 34 h/mo has the lowest frequency.

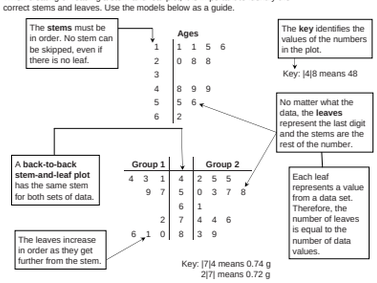
Ⓖ More than half of those who responded spend more than 20 h/mo on the Internet.

Ⓗ Only four people responded that they spend less than 5 h/mo on the Internet.

Ⓙ Sixteen people responded that they spend less than 20 h/mo on the Internet.

READING STRATEGIES

When creating or reading a stem-and-leaf plot, it is important to identify the correct stems and leaves. Use the models below as a guide.



Complete the following.

1. Look at the first stem-and-leaf plot. Which stem has no leaf? **3;**
   List the ages from least to greatest. **11, 11, 15, 16, 20, 28, 28, 48, 49, 49, 55, 56, 62**
2. Look at the back-to-back stem-and-leaf plot.
   Which group has more data values? **Group 2** How many more? **4**
3. What would the stems be if a stem-and-leaf plot were created for the data set:
   39, 52, 67, 68, 71, 75, 84, 84, 86, 88, 91, 100? **3, 4, 5, 6, 7, 8, 9, 10**
   How many leaves would be on the stem labeled 8? **4**
4. The weights, in pounds, of wrestlers competing in a certain weight class at a competition are given below. Create a stem-and-leaf plot of the data. Include a key.
   145, 139, 150, 144, 146, 150, 138, 143, 148, 144, 149, 151, 146, 139, 142

Weights

13	8 9 9
14	2 3 4 4 5 6 6 8 9
15	0 0 1

Key: 14|2 means 142 lb

RETEACH

A **stem-and-leaf plot** arranges data by dividing each data value into two parts: a leaf (the last digit), and a stem (the digit or digits other than the last digit).

The amount of money collected by each student for the drama club is shown below. Use the data to make a stem-and-leaf plot.

55, 82, 90, 113, 100, 90, 93, 68, 66, 108, 116, 56, 85, 89, 102, 103

Step 1: List the stems.

The least value is 55, the greatest value is 116. List stems from 5 to 11. Do not omit any stems.

Step 2: List the leaves.

For each stem, write the ones digit from least to greatest.

Step 3: Write a key explaining one value.

Stems	Leaves
5	5 6
6	6 8
7	
8	2 5 9
9	0 0 3
10	0 2 3 8
11	3 6

Key: 8|2 means 82

The test scores from two different math classes are shown below. Use the data to make a back-to-back stem-and-leaf plot.

Class A: 50, 68, 95, 80, 92, 100, 98, 85, 82, 81
Class B: 75, 81, 100, 63, 52, 94, 100, 100, 87, 99

Step 1: List the stems.

The lowest value is 50, the highest value is 100. List stems from 5 to 10. Do not omit any stems.

Step 2: List the leaves.

For each stem, write the ones digit from least to greatest.

Step 3: Write a key explaining one value from each side.

Leaves (Class A)	Stems	Leaves (Class B)
	5	2
	6	3
	7	5
5 2 1 0	8	1 7
8 5 2	9	4 8
0	10	0 0 0

Key: |8|1 means 81
5|9| means 95

1. The daily low temperatures in degrees Fahrenheit in a town in the Northeast are given below. Use the data to make a back-to-back stem-and-leaf plot.

Daily Low Temperatures (°F)					
40	56	50	60	62	63
49	48	49	40	36	59
57	52	53	42	44	39

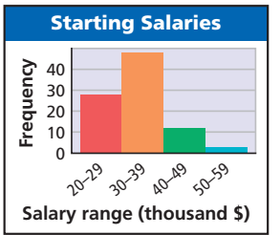
Daily High Temperatures (°F)					
70	84	71	73	71	70
73	78	76	65	65	67
66	76	69	70	70	58

Low Temp		High Temp
	9 6	3
9 9 8 4 2 0 0	4	
9 7 6 3 2 0	5	8
3 2 0	6	5 5 6 7 9
	7	0 0 0 0 1 1 3 3 6 6 8
	8	4

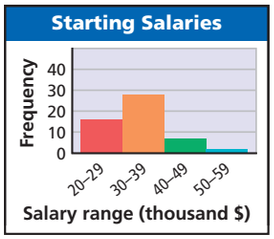
20. The frequencies of starting salary ranges for college graduates are noted in the table. Which histogram best reflects the data?

Starting Salaries	
Salary Range ($)	**Frequency**
20,000–29,000	𝍸 𝍸 𝍸 𝍸 ‖
30,000–39,000	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
40,000–49,000	𝍸 𝍸 𝍸 |
50,000–59,000	|

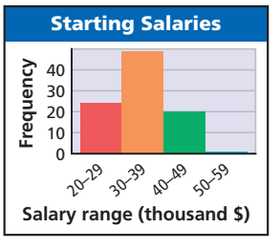
Ⓐ



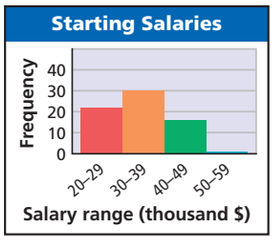
Ⓒ



Ⓑ



Ⓓ



CHALLENGE AND EXTEND

21. The cumulative frequencies of each interval have been given. Use this information to complete the frequency column.

Interval	Frequency	Cumulative Frequency
13–16	8	8
17–20	8	16
21–24	41	57
25–28	66	123

PROBLEM SOLVING

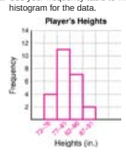
The heights in inches of the 2005 NBA All-Star Game players are given below.

Players' Heights (in.)											
75	78	80	87	72	80	81	83	85	78	76	81
77	78	83	83	78	82	79	80	75	84	82	90

1. Use the data to make a frequency table with intervals. Use an interval of 5.

Players' Heights	
Heights (in.)	**Frequency**
72-76	4
77-81	11
82-86	7
87-91	2

2. Use your frequency table to make a histogram for the data.



Select the best answer.

3. The stem-and-leaf plot below gives the file sizes of the songs on two albums after being converted to mp3 files. What is the largest file size for Album 1?

MP3 File Sizes (megabytes)

Album 1		Album 2
0	5	0 1 2 4 5 5 8
6 6 5 3 1	6	0 6 7
7 3 0	7	3 6 8
3 1	8	6
8 7 2 0	9	9

Key: \| 8 \| 6 means 8.6 MB
1 \| 8 \| means 8.1 MB

A 6.6 MB C 9.8 MB
B 8.9 MB D 9.9 MB

4. The cumulative frequency table below gives the scores of 100 students on a standardized mathematics test. How many students scored between 400 and 499?

Standardized Test Scores	
Scores	**Cumulative Frequency**
200–299	1
300–399	3
400–499	19
500–599	50
600–699	85
700–799	100

F 15 H 19
G 16 J 31

CHALLENGE

You have learned how to create a cumulative frequency table. A graph of a cumulative frequency table is called an **ogive**. Ogives are often presented in terms of percentage.

The test scores of 280 schools are shown. Use the graph to estimate your answers below.

1. What percent of schools had scores less than 1000? 55%
2. How many schools had scores less than 1000? 154
3. What percent of schools scored less than 850? 18%
4. What is the median score? 980
5. Seventy-five percent of schools scored less than what score? 1075
6. A survey was taken of 546 students with jobs. They were asked how many hours they worked per week, on average. Complete the table. Round to the nearest whole percent.

Average Weekly Hours	Frequency	Cumulative Frequency	Cumulative Percentage
0-4	22	22	4%
5-9	40	62	11%
10-14	78	140	26%
15-19	85	225	41%
20-24	90	315	58%
25-29	83	398	73%
30-34	88	486	89%
35-39	52	538	99%
40-44	8	546	100%

7. Draw the ogive.

Use the ogive to answer the following.

8. In which interval does the rate of change increase by the least amount? 40–44
9. Give the range of hours in which the rate of increase appears constant. 10–34

Journal

Have students explain why it is important for a stem-and-leaf plot to have a key.

ALTERNATIVE ASSESSMENT

Have students create or collect a set of data and use that data to make a stem-and-leaf plot, frequency table, and histogram. Encourage students to collect real data.

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10-2 Lesson Quiz

1. The numbers of miles on the new cars in a car lot are given below. Use the data to make a stem-and-leaf plot.
35, 21, 15, 51, 39, 41, 46, 22, 28, 16, 12, 40, 34, 56, 25, 14

New Car Mileage

Stem	Leaves
1	2 4 5 6
2	1 2 5 8
3	4 5 9
4	0 1 6
5	1 6

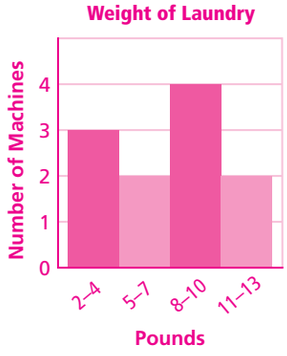
Key: 1\|2 means 12

2. The numbers of pounds of laundry in the washers at a laundromat are given below. Use the data to make a cumulative frequency table.
2, 12, 4, 8, 5, 8, 11, 3, 6, 9, 8

Weight of Laundry in Washers		
Pounds	**Freq.**	**Cum. Freq.**
2–4	3	3
5–7	2	5
8–10	4	9
11–13	2	11

3. Use the frequency table from **Problem 2** to make a histogram.

Weight of Laundry



Also available online

10-3 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Describe the central tendency of a data set.

Create and interpret box-and-whisker plots.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Identify the least and greatest value in each set.

1. 34, 62, 45, 35, 75, 23, 35, 65
   23, 75
2. 1.6, 3.4, 2.6, 4.8, 1.3, 3.5, 4.0
   1.3, 4.8
3. Use the data below to make a stem-and-leaf plot.
   7, 8, 10, 18, 24, 15, 17, 9, 12, 20, 25, 18, 21, 12

Stem	Leaves
0	7 8 9
1	0 2 2 5 7 8 8
2	0 1 4 5

Key: 1\|2 means 12

Also available online

Math Humor

Q: Why would you use the median to describe an average politician?

A: Because he or she is middle-of-the-road.

CC.9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Learn It Online
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COMMON CORE

10-3 Data Distributions

CC.9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *Also* **CC.9-12.S.ID.3, CC.9-12.S.ID.1**

Objectives
Describe the central tendency of a data set.

Create and interpret box-and-whisker plots.

Vocabulary
mean
median
mode
range
outlier
first quartile
third quartile
interquartile range (IQR)
box-and-whisker plot

Who uses this?
Sports analysts examine data distributions to make predictions. (See Example 4.)

A *measure of central tendency* describes the center of a set of data. Measures of central tendency include the *mean*, *median*, and *mode*.

- The **mean** is the average of the data values, or the sum of the values in the set divided by the number of values in the set.
- The **median** is the middle value when the values are in numerical order, or the mean of the two middle numbers if there are an even number of values.
- The **mode** is the value or values that occur most often. A data set may have one mode or more than one mode. If no value occurs more often than another, the data set has no mode.

The **range** of a set of data is the difference between the greatest and least values in the set. The range is one measure of the spread of a data set.

EXAMPLE 1 Finding Mean, Median, Mode, and Range of a Data Set

The numbers of hours Isaac did homework on six days are 3, 8, 4, 6, 5, and 4. Find the mean, median, mode, and range of the data set.

3, 4, 4, 5, 6, 8 — *Write the data in numerical order.*

mean: $\frac{3+4+4+5+6+8}{6} = \frac{30}{6} = 5$ — *Add all the values and divide by the number of values.*

median: 3, 4, 4, 5, 6, 8 — *There is an even number of values. Find the mean of the two middle values.*
The median is 4.5.

mode: 4 — *4 occurs more than any other value.*

range: $8 - 3 = 5$ — *Subtract the least value from the greatest value.*

1. mean: 14 lb; median: 14 lb; modes: 12 lb and 16 lb; range: 4 lb

CHECK IT OUT! **1.** The weights in pounds of five cats are 12, 14, 12, 16, and 16. Find the mean, median, mode, and range of the data set.

A value that is very different from the other values in a data set is called an **outlier**. In the data set below, one value is much greater than the other values.

AP Photo/Dave Martin

1 Introduce

EXPLORATION

Data Distributions

Tori recorded the times, in minutes, it took for her to complete various mazes on her computer.

Times: 10, 15, 13, 23, 80, 19, 17, 15, 15

1. Order the values from least to greatest. 10, 13, 15, 15, 15, 17, 19, 23, 80
2. Determine the difference between the longest time and the shortest time. 70 minutes
3. What time was the most common for Tori to take to complete a maze? 15 minutes
4. When viewing your ordered list from Problem 1, what number appears in the middle of the list? 15
5. What is Tori's average time for completing a maze? 23 minutes
6. If you had to describe Tori's times with a single number, what number would you choose? Why? Possible answer: 15, because it is both the median and the mode.

THINK AND DISCUSS

7. **Discuss** how Tori's average time would change if the time of 80 minutes was removed from the list. If 80 minutes was removed from the list, the average would decrease to 15.9.
8. **Explain** how to predict the number of minutes it will take Tori to complete her next maze. Possible answer: Since 80 appears to be an outlier, use the mean of the times without including 80.

Motivate

Read the following test scores aloud and ask students what value they think should be given as the score of a typical student: 70, 70, 70, 70, 100. Tell students that there are three ways to describe a "typical" score: the value that occurs most frequently, the value in the middle, and the average of the values.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Determining the Effects of Outliers

Identify the outlier in the data set {7, 10, 54, 9, 12, 8, 5}, and determine how the outlier affects the mean, median, mode, and range of the data.

5, 7, 8, 9, 10, 12, 54 *Write the data in numerical order.*

The outlier is 54. *Look for a value much greater or less than the rest.*

With the Outlier:

mean: $\frac{5 + 7 + 8 + 9 + 10 + 12 + 54}{7} = 15$

median: 5, 7, 8, 9, 10, 12, 54
The median is 9.

mode: Each value occurs once.
There is no mode.

range: $54 - 5 = 49$

Without the Outlier:

mean: $\frac{5 + 7 + 8 + 9 + 10 + 12}{6} = 8.5$

median: 5, 7, 8, 9, 10, 12
The median is 8.5.

mode: Each value occurs once.
There is no mode.

range: $12 - 5 = 7$

The outlier increases the mean by 6.5, the median by 0.5, and the range by 42. It has no effect on the mode.

CHECK IT OUT!

2. Identify the outlier in the data set {21, 24, 3, 27, 30, 24}, and determine how the outlier affects the mean, median, mode, and range of the data. **3; the outlier decreases the mean by 3.7 and increases the range by 18. It has no effect on the median and mode.**

As you can see in Example 2, an outlier can strongly affect the mean of a data set, while having little or no impact on the median and mode. Therefore, the mean may not be the best measure to describe a data set that contains an outlier. In such cases, the median or mode may better describe the center of the data set.

EXAMPLE 3 Choosing a Measure of Central Tendency

Niles scored 70, 74, 72, 71, 73, and 96 on his six geography tests. For each question, choose the mean, median, or mode, and give its value.

A **Which measure gives Niles's test average?**

The average of Niles's scores is the mean.

mean: $\frac{70 + 74 + 72 + 71 + 73 + 96}{6} = 76$

B **Which measure best describes Niles's typical score? Explain.**

The outlier of 96 causes the mean to be greater than all but one of the test scores, so it is not the best measure in this situation.

The data set has no mode.

The median best describes the typical score.

median: 70, 71, 72, 73, 74, 96 *Find the mean of the two middle values.*
The median is 72.5.

CHECK IT OUT!

Josh scored 75, 75, 81, 84, and 85 on five tests. For each question, choose the mean, median, or mode, and give its value.

3a. Which measure describes the score Josh received most often? **mode: 75**

3b. Which measure should Josh use to convince his parents that he is doing well in school? Explain. **Median: 81; the median is greater than either the mean or the mode.**

2 Teach

Guided Instruction

Discuss mean, median, mode, and range. Show how an outlier can drastically change the mean and range of a data set, while having little or no effect on the median and mode. Discuss how the presence or lack of an outlier can be used to determine whether a mean or median should be reported as a typical value. Then show how to find first and third quartiles and how to make a box-and-whisker plot.

Reaching All Learners

Through Modeling

Bring 4 students to the front of the class. Give one student 5 pencils, one 2 pencils, one 1 pencil, and one no pencils. Have the students put their pencils in one group and then take turns taking one pencil until none are left. They should now each have 2 pencils, which is the mean. Ask students how this activity connects with the definition of *mean*. The number of pencils each student has at the end is the total number of pencils divided by the number of students.

Power Presentations with PowerPoint®

Additional Examples

Example 1

The weights in pounds of six members of a basketball team are 161, 156, 150, 156, 150, and 163. Find the mean, median, mode, and range of the data set.
mean: 156 lb; median: 156 lb; mode: 150 lb, 156 lb; range: 13 lb

Example 2

Identify the outlier in the data set {16, 23, 21, 18, 75, 21}, and determine how the outlier affects the mean, median, mode, and range of the data.
75; the outlier increases the mean by 9.2 and the range by 52. It has no effect on the median and mode.

Example 3

Rico scored 74, 73, 80, 75, 67, and 54 on his six history tests. For each question, choose the mean, median, or mode, and give its value.

A. Which measure best describes Rico's typical score? Explain.
Median: 73.5; the outlier of 54 lowers the mean, and there is no mode.

B. Which measure should Rico use to describe his test scores to his parents? Explain.
Median: 73.5; the median is greater than the mean, and there is no mode.

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- When is the median a member of the data set?

EXAMPLE 2

- How can you tell without calculating whether a mean will increase or decrease due to an outlier?
- Why will an outlier always have an effect on the range?

EXAMPLE 3

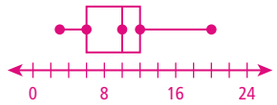
- How does an outlier affect which measure of central tendency you choose?

Power Presentations with PowerPoint®

Additional Examples

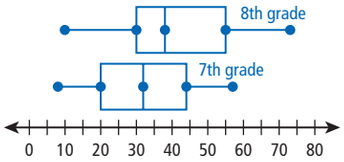
Example 4

The numbers of runs scored by a baseball team in 19 games are given. Use the data to make a box-and-whisker plot.
3, 8, 10, 12, 4, 9, 13, 20, 12, 15, 10, 5, 11, 5, 10, 6, 7, 6, 11



Example 5

The box-and-whisker plots show the number of mugs sold per student in two different grades.



A. About how much greater was the median number of mugs sold by the 8th grade than the median number of mugs sold by the 7th grade? about 6

B. Which data set has a greater maximum? Explain. The data set for the 8th grade; the point representing the maximum is farther to the right for the 8th grade than for the 7th grade.

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- What five values do you need to construct a box-and-whisker plot?
- Why must the median be found before the first and third quartiles?

EXAMPLE 5

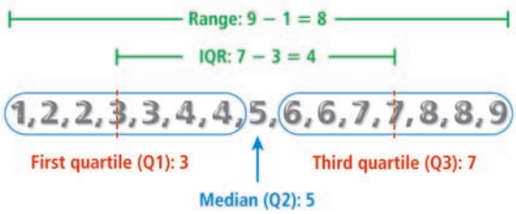
- What is the benefit of using just one number line for both box-and-whisker plots?

Measures of central tendency describe how data cluster around one value. Another way to describe a data set is by its spread—how the data values are spread out from the center.

Quartiles divide a data set into four equal parts. Each quartile contains one-fourth of the values in the set. The **first quartile** is the median of the lower half of the data set. The second quartile is the median of the data set, and the **third quartile** is the median of the upper half of the data set.

Reading Math
The first quartile is sometimes called the lower quartile, and the third quartile is sometimes called the upper quartile.

The **interquartile range (IQR)** of a data set is the difference between the third and first quartiles. It represents the range of the middle half of the data.



A **box-and-whisker plot** can be used to show how the values in a data set are distributed. You need five values to make a box-and-whisker plot: the minimum (or least value), first quartile, median, third quartile, and maximum (or greatest value).

EXAMPLE 4 *Sports Application*

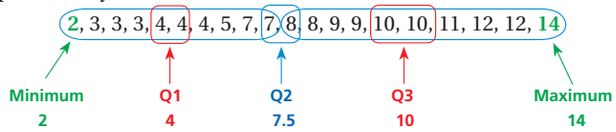
The numbers of runs scored by a softball team in 20 games are given. Use the data to make a box-and-whisker plot.

3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9, 7

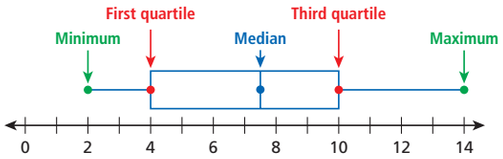
Step 1 Order the data from least to greatest.

2, 3, 3, 3, 4, 4, 4, 5, 7, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

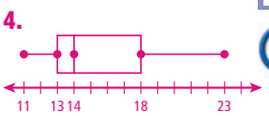
Step 2 Identify the five needed values.



Step 3 Draw a number line and plot a point above each of the five needed values. Draw a box through the first and third quartiles and a vertical line through the median. Draw lines from the box to the minimum and maximum.



4.

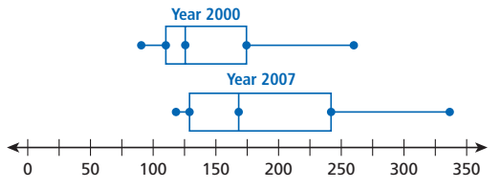


CHECK IT OUT! **4.** Use the data to make a box-and-whisker plot.
13, 14, 18, 13, 12, 17, 15, 12, 13, 19, 11, 14, 14, 18, 22, 23

Teaching Tip **Math Background** One definition of an outlier is any data value that is at least 1.5(IQR) less than the first quartile or at least 1.5(IQR) more than the third quartile.

EXAMPLE 5 Reading and Interpreting Box-and-Whisker Plots

The box-and-whisker plots show the ticket sales, in millions of dollars, of the top 25 movies in 2000 and 2007 (for the United States only).



A **Which data set has a greater median? Explain.**

The vertical line in the box for 2007 is farther to the right than the vertical line in the box for 2000.

The data set for 2007 has a greater median.

B **Which data set has a greater interquartile range? Explain.**

The length of the box for 2007 is greater than the length of the box for 2000.

The data set for 2007 has a greater interquartile range.

C **About how much more were the ticket sales for the top movie in 2007 than for the top movie in 2000?**

2007 maximum: about \$335 million — *Read the maximum values from the box-and-whisker plots.*

2000 maximum: about \$260 million

$335 - 260 = 75$ — *Subtract the maximum values.*

The ticket sales for the top movie in 2007 were about \$75 million more than for the top movie in 2000.

CHECK IT OUT! **Use the box-and-whisker plots above to answer each question.**

5a. Which data set has a smaller range? Explain.

5b. About how much more was the median ticket sales for the top 25 movies in 2007 than in 2000?

5a. The data set for 2000; the distance between the points for the least and greatest values is less for 2000 than for 2007.

5b. about \$40 million

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Explain when the median is a value in the data set.

2. Give an example of a data set for which the mean is twice the median. Explain how you determined your answer.

3. Suppose the minimum in a data set is the same as the first quartile. How would this affect a box-and-whisker plot of the data?

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. Tell which measure of central tendency answers each question.

Measures of Central Tendency	
Measure	**Used to Answer**
	What is the average?
	What is the halfway point of the data?
	What is the most common value?

3 Close

Summarize

Review the steps for creating a box-and-whisker plot.

- Order the data.
- Find the median and the first and third quartiles.
- Draw a number line. Plot the minimum, first quartile, median, third quartile, and maximum, and draw the box and whiskers.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

COMMON ERROR ALERT

Students may include the median in the upper and lower halves of the data set when finding Q1 and Q3. Tell them that the median is never included in these halves.

Teaching Tip **Technology** Students can use a graphing calculator to find the five numbers needed to make a box-and-whisker plot. First enter the data values into a list. Then press STAT and select **1:1-Var Stats** from the **CALC** menu.

Teaching Tip **Visual** If one side of a box-and-whisker plot is longer than the other, it does not mean there are more values on that side. It means that the data values are farther apart from each other on that side.

Answers to *Think and Discuss*

1. Either the data set has an odd number of values, or the data set has an even number of values and the two middle values are the same.
2. Possible answer: 1, 2, 9; start with two numbers and a variable, where the variable is the greatest number in the set: 1, 2, x. The median of this set is 2, so the mean must be 4. Write an equation for the mean: $\frac{1+2+x}{3} = 4$. Solving yields $x = 9$.
3. Because the minimum and the first quartile are the same, the box-and-whisker plot would have no whisker on the left side.
4. See Additional Answers.

Assignment Guide

Assign *Guided Practice* Exercises as necessary.

If you finished Examples **1–3**

- **Basic** 14–21, 27–35, 41–43, 48, 49, 53
- **Average** 14–21, 31–43, 48–53, 58
- **Advanced** 14–21, 28–34 even, 48–53, 58

If you finished Examples **1–5**

- **Basic** 14–35, 41–49, 53–57
- **Average** 14–26, 28–34 even, 36–58
- **Advanced** 14–34 even, 36–60

Homework Quick Check

Quickly check key concepts.
Exercises: 16, 18, 20, 22, 24, 30

Answers

2. mean: 83.75; median: 84; mode: 85; range: 3

3. mean: 31.5; median: 33.5; mode: 44; range: 32

4–8, 10–12, 14–19, 22, 23. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 8–9, 20–21, 24–26, 41–43, 48–49, 51–52, 60

Reason abstractly and quantitatively. Exercises 6–7, 18–19, 36–40

Construct viable arguments and critique the reasoning of others. Exercises 50, 53

Model with mathematics. Exercise 59

Attend to precision. Exercise 35

Learn It Online
State Resources Online

10-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. Vocabulary What is the difference between the *range* and the *interquartile range* of a data set? The range is the difference between the greatest and least values. The interquartile range is the difference between the third and first quartiles.

SEE EXAMPLE 1

Find the mean, median, mode, and range of each data set.

2. 85, 83, 85, 82

3. 12, 22, 33, 34, 44, 44

4. 10, 26, 25, 10, 20, 22, 25, 20

5. 71, 73, 75, 78, 78, 80, 85, 86

SEE EXAMPLE 2

Identify the outlier in each data set, and determine how the outlier affects the mean, median, mode, and range of the data.

6. 10, 96, 12, 17, 15

7. 64, 75, 72, 13, 64

SEE EXAMPLE 3

Adrienne scored 82, 54, 85, 91, and 83 on her last five science tests. For each question, choose the mean, median, or mode, and give its value.

8. Which measure best describes Adrienne's typical score? Explain.

9. Which measure should Adrienne use to convince her soccer coach she is doing well in science? Explain. median: 83; the median is greater than the mean, and there is no mode.

SEE EXAMPLE 4

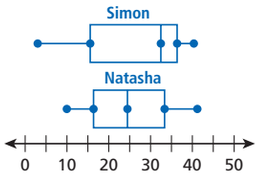
Use the data to make a box-and-whisker plot.

10. 21, 31, 26, 24, 28, 26

11. 12, 13, 42, 62, 62, 82

SEE EXAMPLE 5

The box-and-whisker plots show the scores, in thousands of points, of two players on a video game. Use the box-and-whisker plots to answer each question.



12. Which player has a higher median score? Explain.

13. Which player had the lowest score? Estimate this score. Simon; about 3000 points

PRACTICE AND PROBLEM SOLVING

Find the mean, median, mode, and range of each data set.

14. 75, 63, 89, 91

15. 1, 2, 2, 2, 3, 3, 3, 4

16. 19, 25, 31, 19, 34, 22, 31, 34

17. 58, 58, 60, 60, 60, 61, 63

Identify the outlier in each data set, and determine how the outlier affects the mean, median, mode, and range of the data.

18. 42, 8, 54, 37, 29

19. 3, 8, 3, 3, 23, 8

Lamont bowled 153, 145, 148, and 166 in four games. For each question, choose the mean, median, or mode, and give its value.

20. Which measure gives Lamont's average score? mean: 153

21. Which measure should Lamont use to convince his parents to let him join a bowling league? Explain. mean: 153; the mean is greater than the median, and there is no mode.

Use the data to make a box-and-whisker plot.

22. 62, 63, 62, 64, 68, 62, 62

23. 85, 90, 81, 100, 92, 85

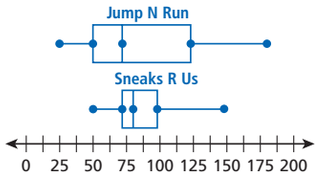
Independent Practice

For Exercises	See Example
14–17	1
18–19	2
20–21	3
22–23	4
24-26	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

The box-and-whisker plots show the prices, in dollars, of athletic shoes at two sports apparel stores. Use the box-and-whisker plots to answer each question.



24. Which store has the greater median price? About how much greater? **Sneaks R Us, about $10**

25. Which store has the smaller interquartile range? What does this tell you about the data sets?

25. Sneaks R Us; the middle half of the data doesn't vary as much at Sneaks R Us as at Jump N Run.

26. Estimate the difference in price between the most expensive shoe type at Jump N Run and the most expensive shoe at Sneaks R Us. **about $32**

Find the mean, median, mode, and range of each data set.

27. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

28. 5, 6, 6, 5, 5

29. 2.1, 4.3, 6.5, 1.2, 3.4

30. $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

31. 23, 25, 26, 25, 23

32. –3, –3, –3, –2, –2, –1

33. 1, 4, 9, 16, 25, 36

34. 1, 0, 0, 1, 1, 4

35. Estimation Estimate the mean of $16\frac{7}{8}$, $12\frac{1}{4}$, $22\frac{1}{10}$, $18\frac{5}{7}$, $19\frac{1}{3}$, $13\frac{8}{11}$, and $13\frac{8}{11}$.
Possible answer: 17

Tell whether each statement is sometimes, always, or never true.

36. The mean is a value in the data set. **sometimes**

37. The median is a value in the data set. **sometimes**

38. If a data set has one mode, the mode is a value in the data set. **always**

39. The mean is affected by including an outlier. **always**

40. The mode is affected by including an outlier. **never**

41. Sports The table shows the attendance at six football games at Jefferson High School. Which measure of central tendency best indicates the typical attendance at a football game? Why?

Attendance at Football Games	
Eagles vs. Bulldogs	743
Eagles vs. Panthers	768
Eagles vs. Coyotes	835
Eagles vs. Bears*	1218
Eagles vs. Colts	797
Eagles vs. Mustangs	854

*Homecoming Game

42. Weather The high temperatures in degrees Fahrenheit on 11 consecutive days were 68, 71, 75, 74, 75, 71, 73, 71, 72, 74, and 79. Find the mean, median, mode, and range of the temperatures. Describe the effect on the mean, median, mode, and range if the next day's temperature was 70 °F.

43. Advertising A home-decorating store sells five types of candles, which are priced at $3, $2, $2, $2, and $15. If the store puts an ad in the paper titled "Best Local Candle Prices," which measure of central tendency should it advertise? Justify your answer.

Use the data to make a box-and-whisker plot.

44. 25, 28, 26, 16, 18, 15, 25, 28, 26, 16

45. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

46. 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4

47. 50, 52, 45, 62, 36, 55, 40, 50, 65, 33

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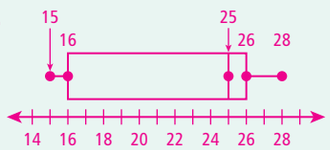
44.


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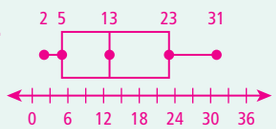

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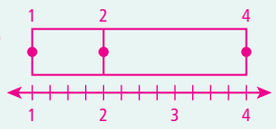

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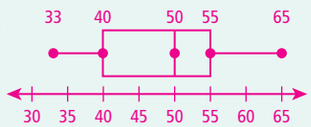


Teaching Tip **Inclusion** In **Exercise 32,** students may expect the range to be a negative number. Use the formula maximum – minimum: $-1 - (-3) = -1 + 3 = 2$.

Answers

27. mean: 5.5; median: 5.5; mode: none; range: 9

28. mean: 5.4; median: 5; mode: 5; range: 1

29. mean: 3.5; median: 3.4; mode: none; range: 5.3

30. mean: $\frac{1}{2}$; median: $\frac{1}{2}$; mode: none; range: 1

31. mean: 24.4; median: 25; modes: 23 and 25; range: 3

32. mean: $-2\frac{1}{3}$; median: $-2\frac{1}{2}$; mode: –3; range: 2

33. mean: $15\frac{1}{6}$; median: $12\frac{1}{2}$; mode: none; range: 35

34. mean: $1\frac{1}{6}$; median: 1; mode: 1; range: 4

41. Median; the mean is affected by the outlier of 1218, and there is no mode.

42. Mean: 73 °F; median: 73 °F; mode: 71 °F; range: 11 °F; the mean would decrease by 0.25 °F, the median would decrease by 0.5 °F, and the mode and range would not change.

43. Median or mode; the store wants their prices to appear low, and the median and mode are both $2.80 less than the mean.

PRACTICE A

PRACTICE C

PRACTICE B

Find the mean, median, mode, and range of each data set.

1. 22, 45, 30, 18, 22
   mean: 27.4 median: 22 mode: 22 range: 27
2. 8, 10, 8, 14, 8, 15
   mean: 10.5 median: 9 mode: 8 range: 7
3. 1.25, 0.5, 3.25, 0.75, 1.75
   mean: 1.5 median: 1.25 mode: none range: 2.75
4. 95, 92, 96, 93, 94, 95, 93
   mean: 94 median: 94 modes: 93, 95 range: 4

Identify the outlier in each data set, and determine how the outlier affects the mean, median, mode, and range of the data.

5. 31, 35, 41, 40, 40, 98
   outlier: 98, increases mean by 10.1 and range by 57, no effect on median or mode
6. 82, 24, 100, 96, 79, 93, 86
   outlier: 24, decreases mean by $9\frac{1}{3}$, median by 3.5, increases range by 55, no effect on mode

7. The amounts of Cathy's last six clothing purchases were $109, $72, $99, $15, $99, and $89. For each question, choose the mean, median, or mode, and give its value.
   a. Which value describes the average of Cathy's purchases? mean: $80.50
   b. Which value would Cathy tell her parents to convince them that she is not spending too much money on clothes? Explain.
   mean, because it is the lowest of the three measures, lower because of the outlier $15
   c. Which value would Cathy tell her parents to convince them that she needs an increase in her allowance? Explain.
   mode, $99, because it is the greatest of the three measures

Use the data to make a box-and-whisker plot.

8. 71, 79, 56, 24, 35, 37, 81, 63, 75
9. 210, 195, 350, 250, 260, 300

The finishing times of two runners for several one-mile races, in minutes, are shown in the box-and-whisker plots.

10. Who has the faster median time? Tim
11. Who has the slowest time? Jamal
12. Overall, who is the faster runner? Explain.
    Tim, his box is to the left of Jamal's.

MULTI-STEP TEST PREP **Exercise 48** involves finding measures of central tendency, including the median. This exercise prepares students for the Multi-Step Test Prep.

Teaching Tip **Number Sense** For **Exercise 49**, ask students if the data seem reasonable and why. If needed, prompt them to think about the manager of the company.

Teaching Tip **Inclusion** Use **Exercise 52** to show the difference between a mode of 0 and no mode.

TEST PREP DOCTOR Tell students who answered **D** in **Exercise 54** that **B** is the *best* answer because, while the range can be found by subtracting, the median is readily available.

Answers

48a. mean: 5.66; median: 5.70; mode: 5.70; range: 0.51

b. 5.96 m

MULTI-STEP TEST PREP

48. The results in Olympic pole-vaulting are given as heights in meters. In the 2008 Olympic Games in Beijing, the following results occurred for the men's pole-vault finals: 5.96, 5.85, 5.70, 5.70, 5.70, 5.70, 5.60, 5.60, 5.60, 5.45, 5.45.

a. Find the mean, median, mode, and range of this data set. Round to the nearest hundredth if needed.

b. The gold medal was won by Steve Hooker of Australia. What was his height in the pole-vault finals?

49. Mean: $32,000, median: $25,000; median; the outlier of $78,000 increases the mean significantly.

49. **Business** The salaries for the eight employees at a small company are shown in the stem-and-leaf plot. Find the mean and median of the salaries. Which measure better describes the typical salary of an employee at this company? Explain.

Salaries ($1000)

Stem	Leaves
2	0 0 3 5 5
3	0 5
4	
5	
6	
7	8

Key: 2\|0 means $20,000

50. **Critical Thinking** Use the data set {1, 2, 3, 5, 8, 13, 21, 34} to complete the following.

a. Find the mean of the data set. **10.875**

b. What happens to the mean of the data set if every number is increased by 2? **The mean increases by 2.**

c. What happens to the mean of the data set if every number is multiplied by 2? **The mean is multiplied by 2.**

51. Allison has taken 5 tests worth 100 points each. Her scores are shown in the grade book below. What score does she need on her next test to have a mean of 90%? **96**

52. 20.75; possible answer: No, Earth's number of moons is much less than either the mean or median number of moons per planet.

53. increase the mean; decrease the mean

Student	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Average
Allison	88	85	89	92	90		
Brad	85	80	87	90	80		

52. **Astronomy** The table shows the number of moons of the planets in our solar system. What is the mean number of moons per planet? Is Earth's value of one moon typical for the solar system? Explain.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Moons	0	0	1	2	63	60	27	13

53. **Write About It** Explain how an outlier with a large value will affect the mean. Explain how an outlier with a small value will affect the mean.

TEST PREP

54. Which value is always represented on a box-and-whisker plot?

Ⓐ Mean Ⓑ Median Ⓒ Mode Ⓓ Range

55. The lengths in feet of the alligators at a zoo are 9, 7, 12, 6, and 10. The lengths in feet of the crocodiles at the zoo are 13, 10, 8, 19, 18, and 16. What is the difference between the mean length of the crocodiles and the mean length of the alligators?

Ⓕ 0.5 foot Ⓖ 5.2 feet Ⓗ 8 feet Ⓙ 11.4 feet

READING STRATEGIES

The mean, median, and mode are measures of central tendency.

	Definition	Example
Mean	the sum of all the values in the data set divided by the number of data values	Data set: {5, 3, 6, 2}; mean: $\frac{5+3+6+2}{4}=\frac{16}{4}=4$
	Effect of an Outlier: Changes significantly	Miscellaneous: Also called the average
Median	Definition: In an ordered data set, it is the middle value, or the mean of the two middle values	Example: Data set: {2, 6, 6, 8, 14, 17, 21}; median: 8
	Effect of an Outlier: Changes either little or not at all	Miscellaneous: Also called the second quartile, Q2, in a box-and-whisker plot
Mode	Definition: The value or values that occur most frequently	Example: Data set: {5, 7, 7, 8, 7, 5, 7, 10}; mode: 7
	Effect of an Outlier: No effect	Miscellaneous: There can be one mode, no mode, or several modes.

Answer each question.

1. Which measure will not give the typical value of the data set when an outlier is present?
   the mean
2. In the data set {1, 1, 2, 2, 2, 3}, why is 1 not a mode?
   because 2 occurs more often
3. When finding a median, when will you have to take the mean of two numbers?
   when there is an even number of data values
4. Why does the adding an outlier to a data set have no effect on the mode?
   The mode occurs most often; an outlier is different from the other numbers, so an outlier cannot be a mode.
5. Find the mean, median, and mode of {7, 3, 10, 15, 10, 3}.
   mean: 8, median: 8.5, mode: 3 and 10

RETEACH

Consider the data set {2, 6, 4, 2, 1}.

The **mean** of the data set is the average of the data set. Add all the numbers and divide by the number of numbers.

$\frac{2+6+4+2+1}{5}=\frac{15}{5}=3$

The mean is 3.

The **median** of the data set is the middle number when the numbers are listed in order.

1, 2, **2**, 4, 6

The median is 2.

If there are two middle numbers, the median is the average of those numbers.

The **mode** is the number that occurs most often.

1, **2**, **2**, 4, 6

The mode is 2.

There can be more than one mode, or there can be no mode.

The **range** is the difference between the greatest and least numbers.

1, 2, 2, 4, **6**

6 – 1 = 5

The range is 5.

Find the mean, median, mode, and range of each data set.

1. 8, 2, 3, 4, 3
   mean: $\frac{8+2+3+4+3}{5}=\frac{20}{5}=4$; mode: **3**
   median: **3**; range: **6**
2. 4, 5, 7, 4, 5, 8
   mean: **5.5**; mode: **4 and 5**
   median: **5**; range: **4**
3. 12, 8, 16, 4
   mean: **10**; mode: **none**
   median: **10**; range: **12**

56. The mean score on a test is 50. Which CANNOT be true?

Ⓐ Half the scores are 0, and half the scores are 100.

Ⓑ The range is 50.

Ⓒ Half the scores are 25, and half the scores are 50.

Ⓓ Every score is 50.

57. Short Response The table shows the weights in pounds of six dogs. How does the mean weight of the dogs change if Rex's weight is not included in the data set?

The mean decreases by 6.6 lb.

Weights of Dogs (lb)			
Duffy	23	Rex	62
Rocky	15	Skipper	34
Pepper	21	Sunny	19

CHALLENGE AND EXTEND

58. List a set of data values with the following measures of central tendency:

mean: 8 median: 7 mode: 6 **Possible answer: 6, 6, 7, 9, 12**

59. Collect a set of data about your classmates or your school. For example, you might collect data about the number of points per game scored by your school's basketball team. Use the data you collect to make a box-and-whisker plot. **Check students' work.**

60. A *weighted average* is an average in which each data value has an importance, or weight, assigned to it. A teacher uses the following weights when determining course grades: homework 25%, tests 30%, and final exam 45%. The table shows Nathalie's scores in the class.

Homework	78, 83, 95, 82, 79, 93
Tests	88, 92, 81
Final exam	90

a. Find the mean of Nathalie's homework scores and the mean of her test scores. **85; 87**

b. Find Nathalie's weighted average for the class. To do so, multiply the homework mean, the test mean, and the final exam score by their corresponding weights. Then add the products. **87.85**

c. What if...? What would Nathalie's mean score for the class be if her teacher did not use a weighted average? **86.1**

PROBLEM SOLVING

Write the correct answer.

1. While window shopping, Sandra recorded the prices of shoes she would like to try on. The prices were $48, $63, $52, $99, and $58. Find the mean, median, and mode of the prices. Which best represents the typical shoe she looked at? Why?

   mean: $64, median: $58, mode: none; The median because only one price is above the mean.

2. The number of cans Xavier recycled each week for eight weeks is 24, 33, 76, 42, 35, 33, 45, and 33. Find the mean, median, and mode of the numbers of cans. How do the mean and median change when the outlier is removed?

   mean: 40.125, median: 34, mode: 33; The mean decreases by 5.125, median decreases by 1.

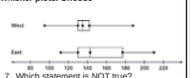
3. The amounts due on the Harvey's electric bill, rounded to the nearest dollar, for the past six months were $64, $83, $76, $134, $76, and $71. Find the mean, median, and mode of the amounts. Which value should Mr. Harvey tell his family to convince them to cut down on electric use?

   mean: 84, median: 76, mode: 76; the mean

4. A manager at a bowling alley surveys adult patrons about their shoes sizes. He records sizes 11, 12, 8, 4, 8, 5, 8, 7, 9, 10, 8, 9, 8, and 10. Find the mean, median, and mode of the sizes. Which is most important to the manager when ordering new rental shoes?

   mean: ≈8.4, median: 8, mode: 8; the mode

The number of traffic citations given daily by two police departments over a two-week period is shown in the box-and-whisker plots. Choose the letter of the best answer.



5. What is the best estimate of the difference in the greatest number of citations given by each department in one day?

   A 10 **B** 20

   C 30 D 35

6. What is the difference in the median number of citations between the two departments?

   F about 8

   G about 15

   H about 22

   J about 40

7. Which statement is NOT true?

   A The East department gave the greatest number of citations in one day.

   B The East department gave the least number of citations in one day.

   C The East department has a greater IQR than the West department.

   D The East department has the greater median number of citations in one day.

CHALLENGE

Box-and-whisker plots can be misleading if either the minimum or maximum value is an outlier. The fair options are either to not use box-and-whisker plots for data with outliers or to show them as individual points.

Mathematically, a value is accepted as an outlier if it is at least 1.5(IQR) below Q1 or 1.5(IQR) above Q3.

For the data set {46, 22, 19, 2, 17, 23, 31, 42, 73, 16, 22, 26, 28}:

1. Find Q1. **18**
2. Find Q3. **36.5**
3. Find the IQR. **18.5**
4. Find 1.5(IQR). **27.75**
5. Find Q1 – 1.5(IQR). **–9.75**
6. Find Q3 + 1.5(IQR). **64.25**
7. Using the rule of being 1.5(IQR) away from Q1 or Q3, what is the outlier in the data set above? **73**

Outliers can be mild or extreme. A **mild outlier** is more than 1.5(IQR) from Q1 or Q3, but less than 3(IQR) from them. An **extreme outlier** is more than 3(IQR) from Q1 or Q3.

8. Classify the outlier identified in question 7. **mild**

To draw a box-and-whisker plot that has outliers, first plot Q1, the median, and Q3 as usual. Draw the box. Plot the minimum and maximum points that are not outliers. Draw the whiskers to these points. Then plot mild outliers with a closed dot and extreme outliers with an open dot.

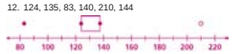
9. Draw a box-and-whisker plot, displaying the outlier, for the data set above.

Draw box-and-whisker plots, displaying outliers, for each data set.

10. 41, 43, 30, 42, 29, 42, 41, 51, 43, 41, 43, 48

11. 30, 28, 27, 29, 33, 10, 21, 32, 27, 29

12. 124, 135, 83, 140, 210, 144



Journal

Have students describe what information a box-and-whisker plot can convey.

ALTERNATIVE ASSESSMENT

Ask students to make up a data set with at least ten values and find the mean, median, mode, and range of the set. Then have them add an outlier and explain the effect that the outlier has on each measure.

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10-3 Lesson Quiz

1. Find the mean, median, mode, and range of the data set {7, 3, 5, 4, 5}.
   mean: 4.8; median: 5; mode: 5; range: 4
2. Identify the outlier in the data set {12, 15, 20, 44, 18, 20}, and determine how the outlier affects the mean, median, mode, and range of the data.
   44; the mean increases by 4.5, median by 1, and range by 24; no effect on mode.
3. The data set {12, 23, 13, 14, 13} gives the times of Tara's one-way ride to school (in minutes) for one week. For each question, choose the mean, median, or mode, and give its value.
   a. Which value describes the time that occurred most often? mode, 13
   b. Which value best describes Tara's ride time? Explain.
   Median, 13 or mode, 13; there is an outlier, so the mean is strongly affected.
4. The amounts of snow (in inches) that fell during the last 8 winters in one city are given. Use the data to make a box-and-whisker plot.
   25, 17, 14, 27, 20, 11, 29, 32
   See Additional Answers.

Also available online

10-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Create dot plots.
Use a dot plot to describe the shape of a data distribution.

Online Edition

Using the Extension

The extension builds on students' knowledge of frequency tables and distributions to introduce dot plots.

Teaching Tip **Kinesthetic** Students can use integer chips, masking tape, and markers to build the dot plots in the examples on their desks.

COMMON CORE

CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Learn It Online
State Resources Online

COMMON CORE

EXTENSION

Dot Plots and Distributions

CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *Also* **CC.9-12.S.ID.1**

A **dot plot** is a data representation that uses a number line and x's, dots, or other symbols to show frequency. Dot plots are sometimes called line plots.

EXAMPLE 1 Making a Dot Plot

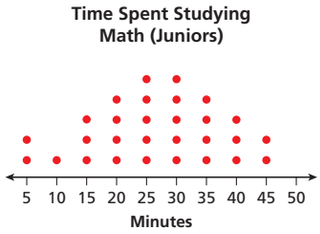
Objectives
Create dot plots.
Use a dot plot to describe the shape of a data distribution.

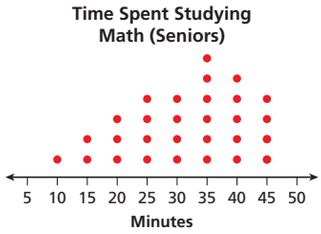
Vocabulary
dot plot
uniform distribution
symmetric distribution
skewed distribution

Mrs. Montoya asked her junior and senior students how many minutes each of them spent studying math in one day, rounded to the nearest five minutes. The results are shown below. Make a dot plot showing the data for juniors and a dot plot showing the data for seniors.

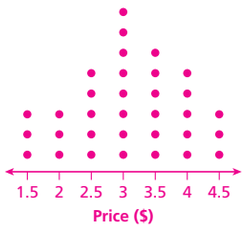
Time Spent Studying Math (min)	Frequency (Juniors)	Frequency (Seniors)
5	2	0
10	1	1
15	3	2
20	4	3
25	5	4
30	5	4
35	4	6
40	3	5
45	2	4

Find the least and greatest values in each data set. Then use these values to draw a number line for each graph. For each student, place a dot above the number line for the number of minutes he or she spent studying.





1. Cafeteria Sales



CHECK IT OUT!

1. The cafeteria offers items at six different prices. John counted how many items were sold at each price for one week. Make a dot plot of the data.

Price ($)	1.50	2.00	2.50	3.00	3.50	4.00	4.50
Items	3	3	5	8	6	5	3

1 Introduce

Motivate

Draw a stem-and-leaf plot on the board. Explain to students that a dot plot looks very similar to a stem-and-leaf plot turned on its side. However, a dot plot does not divide each data value into parts as a stem-and-leaf plot does.

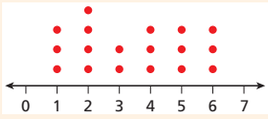
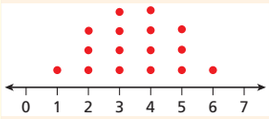
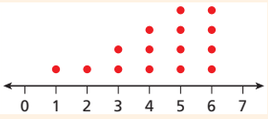
2 Teach

Guided Instruction

Begin the lesson by reviewing frequency tables. Then introduce the new vocabulary, highlighting the connection between frequency tables and dot plots. Once students understand the basic construction of a dot plot, introduce common shapes of distributions and discuss how data can be visually interpreted using a dot plot.

A dot plot gives a visual representation of the distribution, or "shape", of the data. The dot plots in Example 1 have different shapes because the data sets are distributed differently.

Types of Distributions

UNIFORM DISTRIBUTION	SYMMETRIC DISTRIBUTION	SKEWED DISTRIBUTION
		
In a **uniform distribution**, all data points have an approximately equal frequency.	In a **symmetric distribution**, a vertical line can be drawn and the result is a graph divided in two parts that are approximate mirror images of each other.	In a **skewed distribution**, the data is not uniform or symmetric. The data may be skewed to the right or skewed to the left.

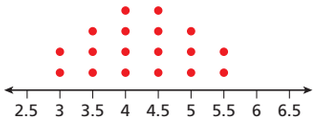
EXAMPLE 2 Shapes of Data Distributions

The data table shows the number of miles run by members of two track teams during one day. Make a dot plot and determine the type of distribution for each team. Explain what the distribution means for each.

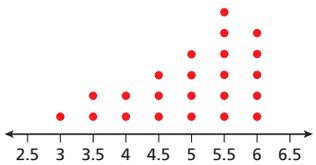
Miles	3	3.5	4	4.5	5	5.5	6
Team A	2	3	4	4	3	2	0
Team B	1	2	2	3	4	6	5

Make dot plots of the data.

Team A



Team B



The data for team A show a symmetric distribution. The distances run are evenly distributed about the mean.

The data for team B show a skewed left distribution. Most team members ran a distance greater than the mean.

CHECK IT OUT!

2. Data for team C members are shown below. Make a dot plot and determine the type of distribution. Explain what the distribution means.

Miles	3	3.5	4	4.5	5	5.5	6
Team C	3	2	2	2	3	2	2

2. Uniform; all team members ran about the same distance.

Teaching Tip **Reading Math** The word *skewed* may be difficult to grasp for some students. Teach synonyms such as *slanted* and *crooked* to help students better understand the word.

3 Close

Summarize

Remind students that dot plots are visual representations of frequency tables. The visual representations may show one of three common types of distributions: skewed, uniform, or symmetric.

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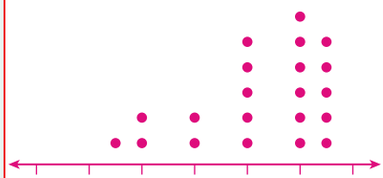
Additional Examples

Example 1 and 2

Gloria is collecting different recipes for chocolate chip cookies. The table shows the cups of flour needed in the recipes. Make a dot plot showing the data. Determine the distribution of the data and explain what the distribution means.

Flour (c)	Number of Recipes
$\frac{3}{4}$	1
1	2
$1\frac{1}{2}$	2
2	5
$2\frac{1}{2}$	6
$2\frac{3}{4}$	5

Amount of Flour in Recipes



The dist. is skewed to the left, which means most recipes require an amount of flour greater than the mean.

Also available online

Mathematical Practices **Questioning Strategies**

EXAMPLE 1

- How can you create a frequency table for qualitative data such as students' favorite colors?

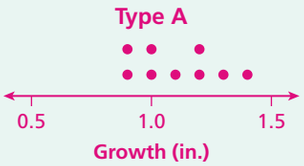
EXAMPLE 2

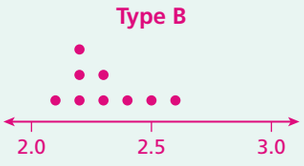
- How do you determine if a data set has no distribution?

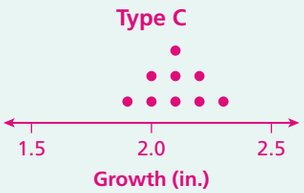
Team C

Answers

1a.





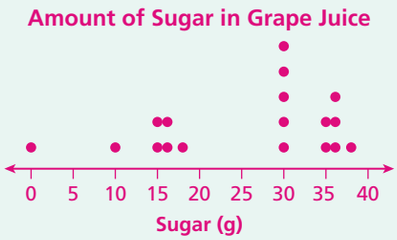


b. A: uniform; B: skewed; C: symmetric

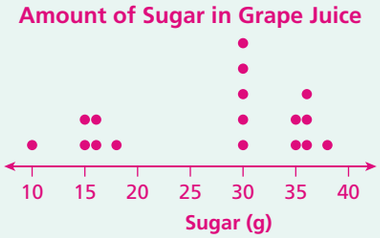
c. A: most: 0.9, 1.0, 1.2; least: 1.1, 1.3. 1.4; B: most: 2.2; least: 2.1, 2.4, 2.5, 2.6; C: most: 2.1; least: 1.9, 2.3

d. A: 1.1, 1.3, 1.4, 0.9, 1.0, 1.2; B: 2.1, 2.4, 2.5, 2.6, 2.3, 2.2; C: 1.9, 2.3, 2.0, 2.2, 2.1

2b. With outlier:

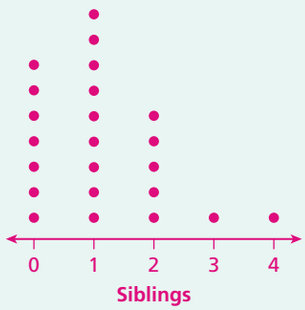


Without outlier:



2c. mean with outlier = 25.3, mean without outlier = 26.8, median = 30; skewed left with or without the outlier

3. Number of Siblings



mean = 1.18, median = 1; skewed right

EXTENSION Exercises

1. Biology Michael is collecting data for the growth of plants after one week. He planted nine seeds for each of three different types of plants and recorded his data in the table below.

Growth of Plants (in.)		
Type A	Type B	Type C
0.9	2.1	1.9
0.9	2.2	2.0
1.0	2.2	2.0
1.0	2.2	2.1
1.1	2.3	2.1
1.2	2.3	2.1
1.2	2.4	2.2
1.3	2.5	2.2
1.4	2.6	2.3

a. Create a dot plot for each type of plant.

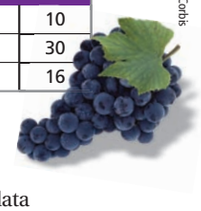
b. Describe the distributions.

c. Which data value(s) occur(s) the most often in each dot plot? the least often?

d. For each dot plot, list the heights in order from least frequent to most frequent.

2. Nutrition Julia researched grape juice brands to determine how many grams of sugar each brand contained per serving (8 fluid ounces = 1 serving). The data she collected is shown in the table.

Grams of Sugar in Grape Juice (per serving)					
15	0	36	18	30	10
30	15	35	30	36	30
36	30	38	16	35	16



© Ocean/Corbis

a. Identify any outlier(s) in the data set. 0

b. Make a dot plot for the data with the outlier(s) and a dot plot for the data without the outlier(s).

c. Describe the distribution of the data with and without the outlier(s).

d. How does excluding the outlier(s) affect the mean, median, and mode of the data set? Mean increases by ≈1.5; median and mode are unchanged.

3. The frequency table shows the number of siblings of each student in a class. Use the table to make a dot plot of the data, and describe the distribution.

Number of Siblings	Frequency
0	7
1	9
2	5
3	1
4	1

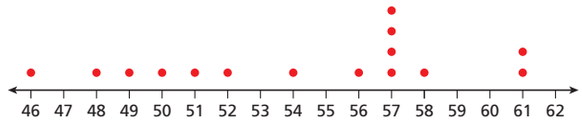
4. **School** The list below shows which grade each member of a high school marching band belongs to.

9, 12, 9, 10, 9, 12, 9, 9, 11, 12, 12, 10, 10, 9, 9, 11, 9, 10, 10, 12, 9, 12, 11, 9, 12, 11, 10, 9, 12, 12, 9, 9, 11, 12

a. Make a dot plot of the data.

b. Explain how you can use the dot plot to find the mean, median, and mode of the data set. Then find each of these values.

Use the dot plot for Exercises 5 and 6.



5. **Write About It** Compare stem-and-leaf plots and dot plots.

a. How are they similar and how are they different?

b. What information can you get from each graph?

c. Can you make a dot plot given a stem-and-leaf plot? Explain.

d. Can you make a stem-and-leaf plot given a dot plot? If so, make a stem-and-leaf plot of the data in the dot plot at right. If not, explain why not.

6. **Write About It** Compare histograms and dots plots.

a. How are they similar and how are they different?

b. What information can you get from each graph?

c. Can you make a dot plot given a histogram? Explain.

d. Can you make a histogram given a dot plot? If so, make a histogram of the data in the dot plot at right. If not, explain why not.

LINK

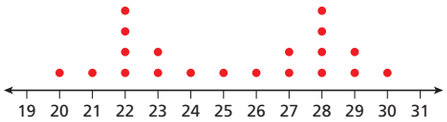
Biology

©Image Source/Getty Images

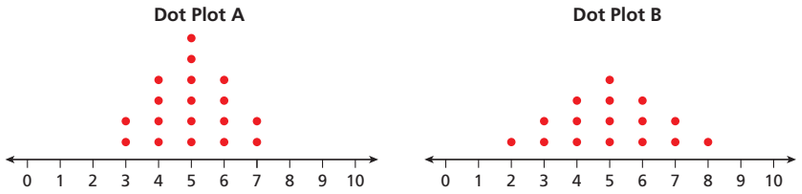
Even though identical twins share the same DNA, they are often of different heights. According to one study, the average height difference between identical twins is 1.7 cm.

7. **Multi-Step** Gather data on the heights of people in your classroom. Separate the data for males from the data for females. Make two dot plots representing the data collected for each group. Compare the dot plots and the distributions of the data.

8. The dot plot at right shows an example of a *bimodal distribution*. Why is this an appropriate name for this type of distribution?



9. **Critical Thinking** Magdalene and Peter conducted the same experiment. Both of their data sets had the same mean. Both made dot plots of their data that showed symmetric distributions, but Peter's dot plot shows a greater range than Magdalene's dot plot. Identify which plot below belongs to Peter and which belongs to Magdalene. **A: Magdalene; B: Peter**

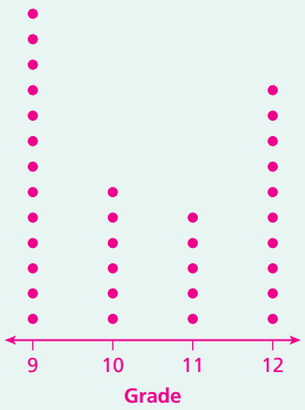


Teaching Tip

Kinesthetic For **Exercise 7**, have students line up according to height in front of the classroom. Create a "life sized" dot plot of height frequencies using students as dots in the dot plot.

Answers

4a. Marching Band Members



b. Mean: multiply each data value by its freq. and divide by the number of data values; ≈10.4

Median: find the data value(s) whose dot(s) are in the middle of the dot plot; 10

Mode: find the data value with the most dots in the dot plot; 9

5a. Both show all of the individual data values, and both show the general shape of the dist. A dot plot shows freq. for individual data values, while a stem-and-leaf plot shows freq. for ranges of data values.

b. You can find mean, med., mode, range, freq., and shape of dist. from both.

c. Yes; a stem-and-leaf plot shows all individual data values.

d. yes

Stem	Leaves
4	6 8 9
5	0 1 2 4 6 7 7 7 7 8
6	1 1

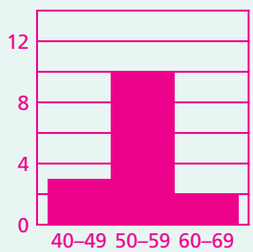
Key: *4|6 means 46.*

6a. Both show freq., but a dot plot shows freq. for individual data values, while a histogram shows freq. for ranges of data values. They are visually similar and show the general shape of the dist. A dot plot shows all of the individual data values, but a histogram does not.

b. Dot plot: individual data values, mean, med., mode, range, freq., shape of dist; histogram: shape of dist.

c. No; a histogram does not show all of the individual data values.

6d.



7. Check students' work.

8. The data set has two modes (in this case, 22 and 28)

Technology Organizer

Use with the lesson *Data Distributions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a spreadsheet or graphing calculator to create graphs.

Materials: spreadsheet program, graphing calculator

Online Edition
TechKeys, Graphing Calculator

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

In **Activity 1,** the spreadsheet program will automatically choose a reasonable scale and interval for the data. However, these can be changed by clicking on the vertical axis of the bar graph and selecting the Scale tab.

The legend was turned off to create a bar graph. The legend should be turned on for double-bar graphs and double-line graphs. The legend should also be turned on when the sectors of a circle graph are not individually labeled.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Learn It Online
State Resources Online

10-3 Technology LAB

Use with Data Distributions

Use Technology to Make Graphs

You can use a spreadsheet program to create bar graphs, line graphs, and circle graphs. You can also use a graphing calculator to make a box-and-whisker plot.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

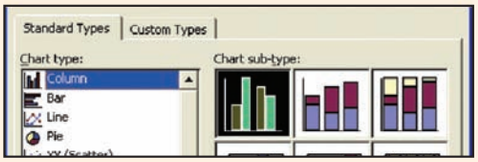
Learn It Online
Lab Resources Online

Activity 1

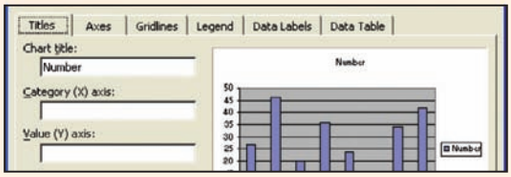
Many colors are used on the flags of the 50 United States. The table shows the number of flags that use each color. Use a spreadsheet program to make a bar graph to display the data.

Color	Black	Blue	Brown	Gold	Green	Purple	Red	White
Number	27	46	20	36	24	4	34	42

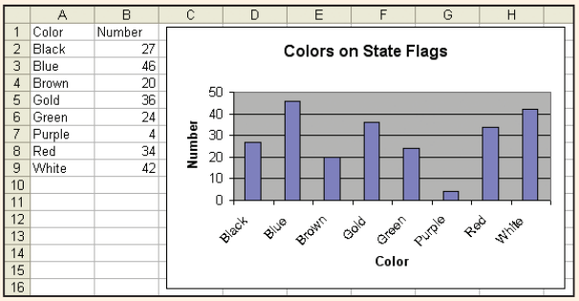
1. Enter the data from the table in the first two columns of the spreadsheet.
2. Select the cells containing the titles and the data.
   Then click the Chart Wizard icon, . Click Column from the list on the left, and then choose the small picture of a vertical bar graph. Click Next.



3. The next screen shows the range of cells used to make the graph. Click Next.
4. Give the chart a title and enter titles for the *x*-axis and *y*-axis. Click the Legend tab, and then click the box next to Show Legend to turn off the key. (A key is needed when making a double-bar graph.) Click Next.



5. Click Finish to place the chart in the spreadsheet.



Alternative Approach

In **Activity 2,** have students select the fourth type of graph, which will show marks at outliers. Discuss why there are outliers in the data set and how they affect the five important values.

Try This

1. The table shows the average number of hours of sleep people at different ages get each night. Use a spreadsheet program to make a bar graph to display the data.

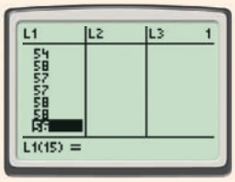
Age (yr)	3–9	10–13	14–18	19–30	31–45	46–50	51+
Sleep (h)	11	10	9	8	7.5	6	5.5

Activity 2

Adrianne is a waitress at a restaurant. The amounts Adrianne made in tips during her last 15 shifts are listed below. Use a graphing calculator to make a box-and-whisker plot to display the data. Give the minimum, first quartile, median, third quartile, and maximum values.

$58, $63, $40, $44, $57, $59, $61, $53, $54, $58, $57, $57, $58, $58, $56

1. To make a list of the data, press STAT, select **Edit**, and enter the values in List 1 **(L1)**. Press ENTER after each value.



2. To use the **STAT PLOT** editor to set up the box-and-whisker plot, press 2nd STAT PLOT Y=, and then ENTER.

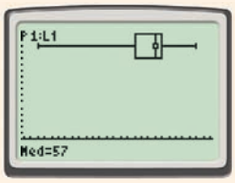
   Press ENTER to select **Plot 1**.

3. Select **On**. Then use the arrow keys to choose the fifth type of graph, a box-and-whisker plot.

   Xlist should be **L1** and **Freq:** should be 1.

4. Press ZOOM and select **9: ZoomStat** to see the graph in the statistics window.

5. Use TRACE and the arrow keys to move the cursor along the graph to the five important values: minimum **(MinX)**, first quartile **(Q1)**, median **(MED)**, third quartile **(Q3)**, and maximum **(MaxX)**.



   minimum: 40

   first quartile: 54

   median: 57

   third quartile: 58

   maximum: 63

Try This

2. The average length in inches of the ten longest bones in the human body are listed. Use a graphing calculator to make a box-and-whisker plot to display the data. What are the minimum, first quartile, median, third quartile, and maximum values of the data set?

   19.88, 16.94, 15.94, 14.35, 11.10, 10.40, 9.45, 9.06, 7.28, 6.69

Close

Key Concept

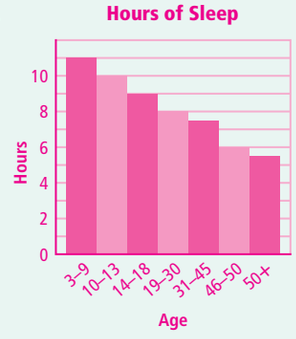
Spreadsheets can be used to quickly create several types of data displays, including bar graphs, line graphs, and circle graphs. Box-and-whisker plots can be created on a graphing calculator.

Assessment

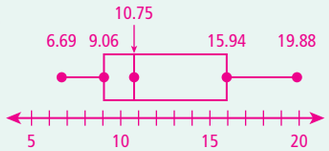
Journal Have students explain how to create and interpret a box-and-whisker plot on a graphing calculator.

Answers to *Try This*

1.



2.



min.: 6.69; Q1: 9.06; med.: 10.75; Q3: 15.94; max.: 19.88

10-4 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Recognize misleading graphs.

Recognize misleading statistics.

Online Edition
Interactivity, Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Determine the measure of central tendency that best describes each set of data.

1. home team scores for basketball games: 56, 61, 75, 49, 69, 91, 87, 54, 79, 62 mean
2. number of scouts in the city's scouting troops: 10, 8, 21, 11, 9, 7, 5, 10, 6, 9 median
3. number of apricots in bags of mixed dried fruit: 3, 3, 3, 4, 3, 3, 2, 0, 3, 3, 4, 6, 3, 2, 3, 3 mode

Also available online

Math Humor

Q: What is the most misleading type of graph?

A: A lyin' graph.

COMMON CORE

CC.9-12.S.IC.6 Evaluate reports based on data.

Learn It Online
State Resources Online

COMMON CORE

10-4 Misleading Graphs and Statistics

CC.9-12.S.IC.6 Evaluate reports based on data.

Objectives
Recognize misleading graphs.

Recognize misleading statistics.

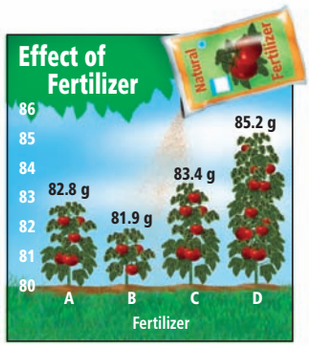
Vocabulary
random sample

Why learn this?
A misleading graph can be used to distort the results of a student council election. (See Example 3.)

Graphs can be used to influence what people believe. The way a data set is displayed can influence how the data are interpreted.

EXAMPLE 1 Misleading Bar Graphs

The graph shows the size of tomatoes on plants that were treated with different fertilizers.



A **Explain why the graph is misleading.**
The scale on the vertical axis begins at 80. This exaggerates the differences between the sizes of the bars.

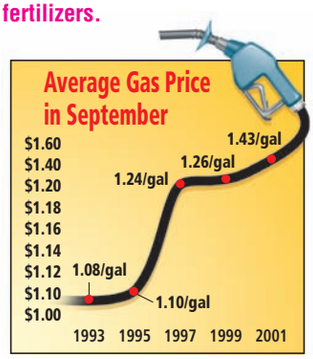
B **What might someone believe because of the graph?**
Someone might believe that the tomato treated with fertilizer D is much larger than the other tomatoes. It is only 3.3 grams larger than the tomato treated with fertilizer B.

CHECK IT OUT!

1. Who might want to use the graph above? Explain.
Company D; the fertilizer from company D appears to be more effective than the other fertilizers.

EXAMPLE 2 Misleading Line Graphs

The graph shows the average price of gasoline in the U.S. in September.



A **Explain why the graph is misleading.**
The intervals on the vertical axis are not equal.

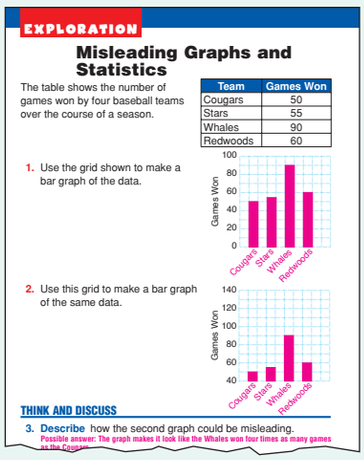
B **What might people be influenced to believe by the graph?**
Someone might believe that the price of gasoline increased the most between 1995 and 1997. However, the change between 1995 and 1997 was only \$0.14/gal while the change between 1999 and 2001 was \$0.17/gal.

2. Possible answer: taxi drivers; the drivers could justify charging higher rates by using this graph, which seems to show that gas prices have increased dramatically.

CHECK IT OUT!

2. Who might want to use the graph above? Explain.

1 Introduce


EXPLORATION

Misleading Graphs and Statistics

The table shows the number of games won by four baseball teams over the course of a season.

Team	Games Won
Cougars	50
Stars	55
Whales	90
Redwoods	60

1. Use the grid shown to make a bar graph of the data.

2. Use this grid to make a bar graph of the same data.

THINK AND DISCUSS

3. **Describe** how the second graph could be misleading.
Possible answer: The graph makes it look like the Whales won four times as many games as the Cougars

Motivate

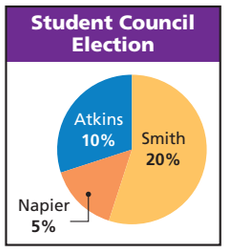
Discuss with students some situations in which someone might want to make a graph that suggests an incorrect conclusion. Examples might include companies who want to sell their product, candidates who want votes, and so on.

Explorations and answers are provided in your online resources.

A circle graph compares each category of a data set to the whole. When any category is not represented in the graph, it may appear that another category represents a greater percentage of the total than it should.

EXAMPLE 3 Misleading Circle Graphs

The graph shows what percent of the total votes were received by three candidates for student council president.



A **Explain why the graph is misleading.**
The sections of the graph do not add to 100%, so the votes for at least one of the candidates is not represented.

B **What might people be influenced to believe by the graph?**
Someone might believe that Smith won the election.

CHECK IT OUT!

3. Who might want to use the graph above? Explain.
Possible answer: Smith; Smith might want to show that he or she got many more votes than Atkins or Napier.

Statistics can be misleading because of the way the data is collected or the way the results are reported. A sufficiently large *random sample* is a good way to collect unbiased data. In a **random sample**, all members of the group being surveyed have an equal chance of being selected.

EXAMPLE 4 Misleading Statistics

A researcher surveys people leaving a basketball game about what they like to watch on TV. Explain why the following statement is misleading: "80% of people like to watch sports on TV."

The sample is biased because people who attend sporting events are more likely to watch sports on TV than people who watch TV but do not attend sporting events.

CHECK IT OUT!

4. A researcher asks 4 people if they have seasonal allergies. Three people respond yes. Explain why the following statement is misleading: "75% of people have seasonal allergies."
The sample size is much too small.

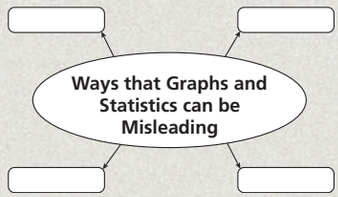
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Give an example of a situation in which someone might intentionally try to make a graph misleading.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Add more boxes if needed.



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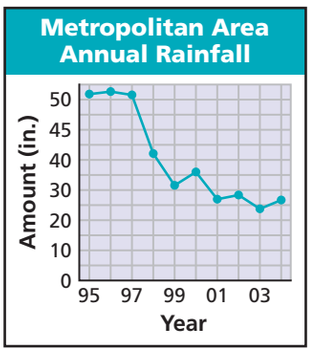
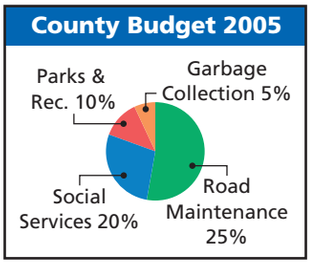
Additional Examples

Examples 1–3





A. Explain why each graph is misleading.

B. What might someone believe because of each graph?

Example 4

A study of 5 households found the following number of pets per household: 2, 1, 1, 9, and 2. Explain why the following statement is misleading: "The average household has 3 pets."
For answers, see Additional Answers.

Also available online

2 Teach

Guided Instruction

Review the characteristics of line, bar, and circle graphs. In **Example 3,** tell students that another way for circle graphs to be misleading is if the sectors are not drawn to scale. If a sector representing 20% of the data is larger than a sector representing 50% of the data, the graph is misleading, even if the percents add to 100%.

Teaching Tip **Inclusion** Have students redraw each of the graphs in the examples so that they are no longer misleading. (Have them create more data for the circle graph.)

3 Close

Summarize

A misleading graph may cause a certain interpretation of the data. Tell students to check that the scale fits the data so that relationships are not exaggerated or minimized. The scale should also have evenly spaced intervals.

Answers to *Think and Discuss*

1. Possible answer: Someone selling a product might want to make the product appear better than other similar products.
2. See Additional Answers.

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLES 1–3

- How can an accurate graph or statement about data be misleading?

EXAMPLE 4

- Why is a random sample the best way to collect accurate data?

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–4**

Basic 7–12, 15, 16
Average 7–17
Advanced 7–11, 13–18

Homework Quick Check
Quickly check key concepts.
Exercises: 7–10, 13

Teaching Tip **Reading Math** In part **c** of **Exercises 2–4,** students may find it easier to answer the question, "Who would profit from this graph?"

Answers

1. No member of a group is more likely to be picked for the sample than any other member of the group.

2a. The vert. scale does not start at 0. This exaggerates the differences in heights of the bars.

b. Someone might believe that employees at company Y make about twice as much as employees at company Z.

3c. Possible answer: someone who wants to increase spending on promoting tourism in San Francisco; the graph makes it appear that San Francisco is becoming a less popular travel destination.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 2–11, 17–18

Construct viable arguments and critique the reasoning of others. Exercises 12–14

Learn It Online
State Resources Online

10-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

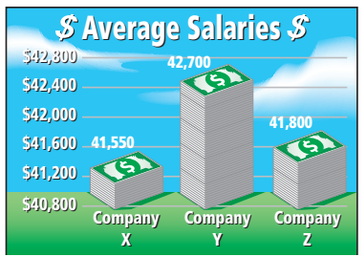
GUIDED PRACTICE

1. **Vocabulary** Explain in your own words what the term *random sample* means.

SEE EXAMPLE 1

2. The graph shows the average salaries of employees at three companies.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

Possible answer: company Y; the graph makes it appear that company Y pays high salaries.

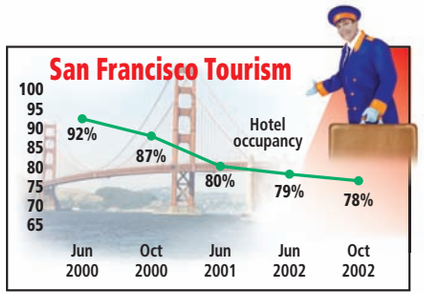


SEE EXAMPLE 2

3. The graph shows hotel occupancy in San Francisco over four years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

3a. The vert. scale does not start at 0, and the categories on the horiz. scale are not at equal time intervals.

3b. Tourism is decreasing rapidly.



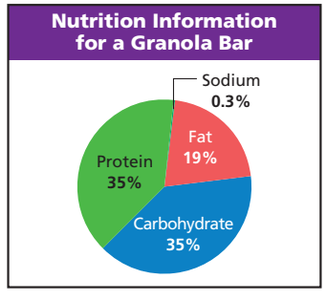
SEE EXAMPLE 3

4. The graph shows the nutritional information for a granola bar.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

4a. The sectors of the graph do not add to 100%.

b. Possible answer: The granola bar is almost half protein.

Possible answer: the granola bar company; they might want their product to look healthful.



SEE EXAMPLE 4

5. Three students were surveyed about their favorite teacher. Two students answer Mr. Gregory, and one answers Mr. Blaine. Explain why the following statement is misleading: "Mr. Gregory is the favorite teacher of a majority of the students." **The sample size is too small.**

6. A researcher surveys people at a shopping mall about whether they favor enlarging the size of the mall parking lot. Explain why the following statement is misleading: "85% of the community is in favor of enlarging the parking lot."
People shopping at the mall are more likely to favor a larger parking lot.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

PRACTICE AND PROBLEM SOLVING

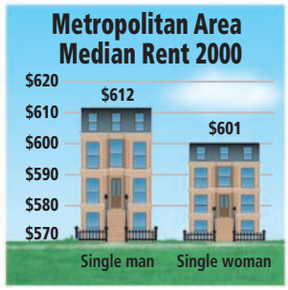
Independent Practice

For Exercises	See Example
7	1
8	2
9	3
10	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

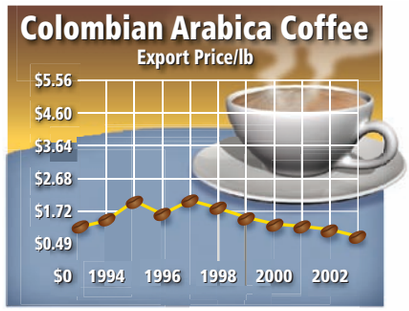
7. The graph shows the median rent for men and women in a metropolitan area.



- **a.** Explain why the graph is misleading.
- **b.** What might someone believe because of the graph?
- **c.** Who might want to use this graph? Explain.

Possible answer: single men looking for apartments who want to negotiate a lower rent; the graph makes it appear that single men are overcharged.

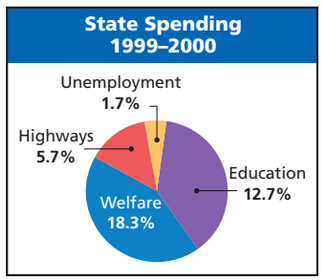
8. The graph shows the export prices of Colombian arabica coffee over nine years.



- **a.** Explain why the graph is misleading.
- **b.** What might someone believe because of the graph?
- **c.** Who might want to use this graph? Explain.

Possible answer: coffee growers; the graph makes it appear that coffee prices are low.

9. The graph shows how the state spent tax dollars during 1999 and 2000.



- **a.** Explain why the graph is misleading.
- **b.** What might someone believe because of the graph?
- **c.** Who might want to use this graph? Explain.

10. A college math course has one section with 240 students and 8 sections with 30 students. Explain why the following statement is misleading: "The average class size for the course is 53 students."

Most of the classes have fewer than 53 students. The mean is not a good descriptor.

MULTI-STEP TEST PREP

11. The table shows scores from the women's gymnastics finals in the floor exercise at the 2004 Summer Olympic Games.

Rank	Name	Score
1	Catalina Ponor	9.750
2	Nicoleta Sofronie	9.562
3	Patricia Moreno	9.487
4	Fei Cheng	9.412
5	Daiane dos Santos	9.375
6	Mohini Bhardwaj	9.312
7	Kate Richardson	9.312
8	Alina Kozich	8.500

- **a.** Find the average score for the women in the finals. 9.339
- **b.** Would it be misleading to describe the data using the average score? Explain.
- **c.** Make a graph for this data that could convince someone that the difference between the first place score and the eighth place score was very small.

b. Yes; the lowest score, 8.500, brings the mean lower than most of the scores.

COMMON ERROR ALERT

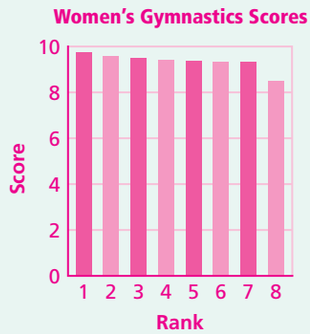
In **Exercise 3,** students may look only at the vertical scale. Point out that the graph shows data for months that are not evenly spaced on the horizontal scale. Also, measuring hotel occupancy at different times of the year may not be an accurate way to gauge tourism.

MULTI-STEP TEST PREP **Exercise 11** involves finding the mean of a data set and determining whether it is the best statistic to describe the data set. This exercise prepares students for the Multi-Step Test Prep.

Answers

9c. Possible answer: someone who wants to justify cutting spending on welfare; the graph makes it appear that a large part of the spending was for welfare.

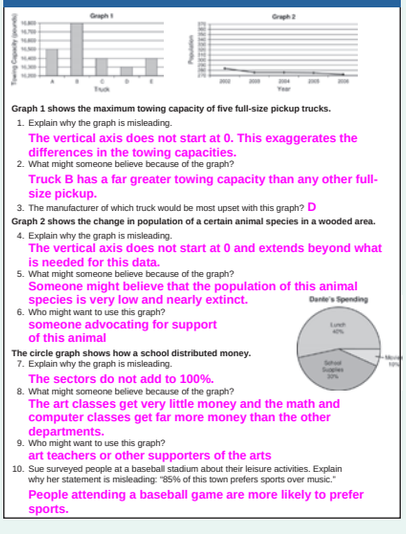
11c.



PRACTICE A

PRACTICE C

PRACTICE B



Graph 1 shows the maximum towing capacity of five full-size pickup trucks.

1. Explain why the graph is misleading.
   The vertical axis does not start at 0. This exaggerates the differences in the towing capacities.
2. What might someone believe because of the graph?
   Truck B has a far greater towing capacity than any other full-size pickup.
3. The manufacturer of which truck would be most upset with this graph? D

Graph 2 shows the change in population of a certain animal species in a wooded area.

4. Explain why the graph is misleading.
   The vertical axis does not start at 0 and extends beyond what is needed for this data.
5. What might someone believe because of the graph?
   Someone might believe that the population of this animal species is very low and nearly extinct.
6. Who might want to use this graph?
   someone advocating for support of this animal

The circle graph shows how a school distributed money.

7. Explain why the graph is misleading.
   The sectors do not add to 100%.
8. What might someone believe because of the graph?
   The art classes get very little money and the math and computer classes get far more money than the other departments.
9. Who might want to use this graph?
   art teachers or other supporters of the arts
10. Sue surveyed people at a baseball stadium about their leisure activities. Explain why her statement is misleading: "85% of this town prefers sports over music."
    People attending a baseball game are more likely to prefer sports.

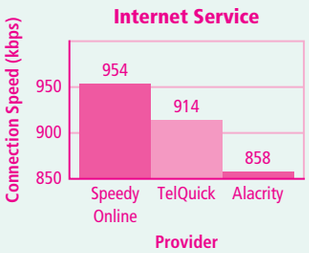
Teaching Tip **Technology** For **Exercise 13,** students can easily change the scale of the vertical axis with graph-making software or a spreadsheet program. The defaults for these programs usually generate relatively accurate graphs. By adjusting the preferences for the vertical scale, students can generate graphs for parts **b** and **c.**

TEST PREP DOCTOR Students who answered **C** for **Exercise 15** may have seen that the mortality rate greatly decreased over time. Point out that most of the decrease occurred before the vaccine was introduced in 1963.

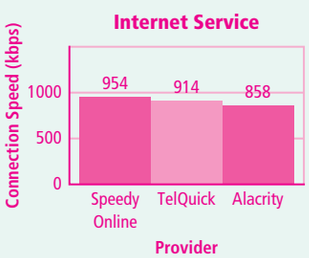
Teaching Tip **Social Studies** **Exercise 16** is about voting in the United States and France. Some students may not have a strong understanding of the political process in either country. Explain that in both countries, adults who meet certain basic requirements (citizenship, for example) may register to vote in elections. Both countries are democracies.

Answers

13a.



b.

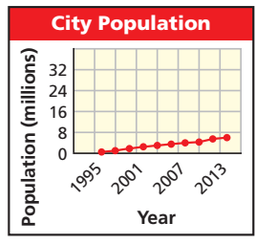


c. Possible answer: an objective report or Alacrity advertising materials. Someone who has no reason to make one company look better than another might make this graph. Alacrity might make this graph to show that they are very similar to the other companies.

17. See Additional Answers.

12. B; the vert. scale extends beyond what is needed, so it appears that the pop. grew slowly; however, the pop. is several times the size that it started.

12. ///**ERROR ANALYSIS**/// The graph shows the population of a city over time. Which conclusion is incorrect? Explain why the conclusion is incorrect and how the graph was misleading.



A The population has grown rapidly over 20 years.

B The population has grown slowly over 20 years.

13. The table shows the average connection speeds of some broadband Internet service providers.

Provider	Connection Speed (Kbps)
Speedy Online	954
TelQuick	914
Alacrity	858

a. Construct a display that suggests that Speedy Online is much faster than the other services.

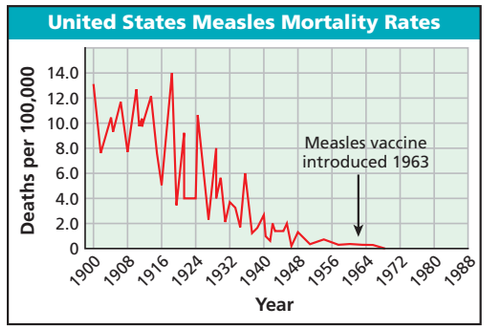
b. Construct a display that suggests that all of the services offer about the same connection speeds.

c. Write About It Where might you expect to see your graph from part **b**? Explain.

14. Critical Thinking Explain how a graph can show truthful data but still be misleading. **Possible answer: The scale of a graph can exaggerate the differences in the heights of bars or the slopes of lines. The data may be accurate, but the display encourages an inaccurate interpretation.**

TEST PREP

15. What might someone be influenced to believe because of the graph?



Source: www.healthsentinel.com

(A) The measles vaccine was introduced when the mortality rate was at its highest.

(B) The measles vaccine was unnecessary.

(C) The measles vaccine dramatically decreased the mortality rate.

(D) The measles vaccine increased the mortality rate.

16. The table shows the number of votes cast in the 2000 U.S. presidential election and in the 2002 French presidential election. What additional information is needed to determine whether the following statement is misleading?

Country	Votes Cast
United States	105,405,100
France	29,497,272

"American voters are more likely to vote than French voters."

(F) The number of candidates in each election

(G) The legal voting age in France

(H) The number of registered voters in the United States in 2000 and France in 2002

(J) The number of polling locations in the United States in 2000 and France in 2002

READING STRATEGIES

There are several ways for graphs and statistics to be misleading.

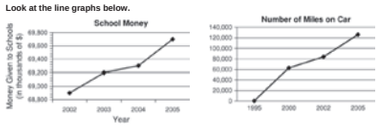
Graphs
- The scale on the vertical axis does not start at 0, or extends beyond what is needed.
- The intervals on the horizontal or vertical axis are not equal.
- The sectors of a circle graph do not total 100%.

Statistics
- The sample size is too small to be representative.
- The population surveyed has a bias toward the topic.
- The mean is used to represent data that has an outlier.

Answer the following.

1. When is it misleading to use the mean as a descriptor of a data set?
   when there is an outlier in the data set
2. What should you check when analyzing a circle graph?
   check if the percents add to 100%
3. Which would be more trustworthy: a survey of 10 people or a survey of 100 people?
   a survey of 100 people

Look at the line graphs below.

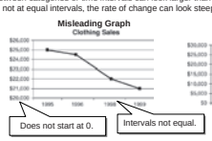


4. How is the first graph misleading? What might someone believe because of it?
   The vertical axis starts and ends just beyond the data values. Someone might believe there's been a big increase in money given to schools.
5. How is the second graph misleading? What might someone believe because of it?
   The horizontal axis is not equally spaced. Someone might believe a lot of miles were put on very quickly when the car was new.

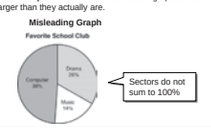
RETEACH

Graphs can be used to mislead people.

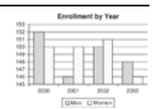
Bar Graphs and Line Graphs: If the vertical scale does not start at 0, the difference between categories or time intervals can look larger than it is. If the horizontal scale is not at equal intervals, the rate of change can look steeper than it is.



Circle Graphs: If the sections in a circle graph do not sum to 100%, sections will appear larger than they actually are.



1. The graph shows the number of men and women who have enrolled in a school.
   a. Explain why the graph is misleading.
      The vertical scale does not start at zero.
   b. What might someone believe because of the graph?
      Someone may believe there are large differences between the number of men and women who have enrolled at the school.
   c. Who might want to use this graph?
      Possible answer: Someone who may want to show that inequalities exist in admissions.



CHALLENGE AND EXTEND

17. Logic A fingerprint analyst is studying a fingerprint that was found in the chemistry lab. He reports that the fingerprint belongs to Dr. Arenson. Below are two questions the analyst was asked and the answers he gave.

Question 1: What are the chances that the fingerprint belongs to someone else who has the same fingerprint as Dr. Arenson?

Answer: One in several billion.

Question 2: What are the chances that the fingerprint was wrongly identified?

Answer: About 1 in 100.

a. What is the difference between the two questions?

b. What does the answer to question 1 lead you to believe?

c. Who do you think might have asked question 1?

d. What does the answer to question 2 lead you to believe?

e. Who do you think might have asked question 2?

LINK **History**

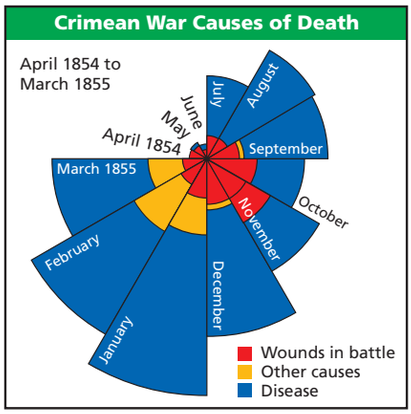
Florence Nightingale (1820–1910) served as a nurse in the Crimean War. In 1854, she brought the first female nurses to military hospitals.

18. History Graphs like the one at right were created by Florence Nightingale. Nightingale served as a nurse during the Crimean War and was concerned with the unsanitary conditions the soldiers lived in. Each "wedge" of the circle represents a month between April 1854 and March 1855.

a. What do you think Florence Nightingale wanted to show with this graph?

b. Who do you think Nightingale showed the graph to?

18a. Possible answer: Most deaths during the Crimean War were due to unclean conditions.
b. Possible answer: military officials who could control the conditions of military hospitals



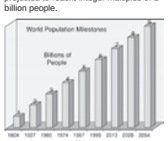
Source: The Florence Nightingale Museum

© Bettmann/CORBIS

PROBLEM SOLVING

Analyze these misleading graphs.

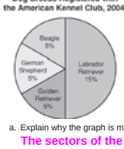
1. This graph shows the years in which the world population reached, or is projected to reach, integer multiples of 1 billion people.



a. Explain why the graph is misleading.
The years are not at equal time intervals.

b. What might someone believe because of the graph?
that world population is increasing in a linear fashion

2. This graph shows dog registrations with the American Kennel Club.



a. Explain why the graph is misleading.
The sectors of the graph do not add to 100% because several dog breeds are omitted.

b. Who might want to use this graph, and why?
A breeder of labrador retrievers might use the graph to overemphasize the popularity of the breed.

Use the graph to select the best answer.

3. The graph is misleading because of which scale(s)?
A horizontal only C both
B vertical only D neither

4. Which statement is certainly true?
F The average house price increased every year between 1995 and 2004.
G The average house prices have increased $80,000 between 1995 and 2004.
H The average house price in 2001 was about $160,000.
J None of the above.

CHALLENGE

In order to make bar graphs more interesting, pictures are often used to replace the bars. However, this sometimes results in misleading graphs.

This graph shows the number and type of pets owned by the students in a fourth-grade class. Each picture represents one animal. (*Note:* When pictures are used to count items in this manner, the graph is frequently called a *pictograph*.)

1. Explain why the graph is misleading.
The pictures are different sizes, so the numbers of some pets are exaggerated.

2. What might someone believe because of the graph?
There is the same number of cats as horses; and there are more dogs and fewer fish than any other animal.

3. Who might want to use this graph? Why?
A breeder of dogs; to suggest that dogs are the most popular pet.

4. How could you improve the graph?
Make the individual pictures the same sizes, or use a standard bar graph.

This graph shows the number of computer models, by brand, that are sold at a store.

5. Explain why the graph is misleading.
Because it is 3-D, the relative numbers are exaggerated by the volumes.

6. What might someone believe because of the graph?
The number of Brand A models is the number of Brand C models.

7. Who might want to use this graph? Why?
The manufacturer of Brand A; to suggest that their models offer significantly more variety.

8. How could you improve the graph?
Use 2-dimensional bars that only vary by height, not by width and/or depth.

Journal

Have students write four questions they could ask themselves to determine whether a graph or description of data is misleading.

ALTERNATIVE ASSESSMENT

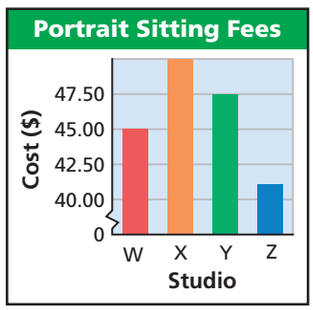
Have students create misleading bar, line, and circle graphs. Then have them explain why their graphs are misleading and draw accurate representations.

Power Presentations with PowerPoint®

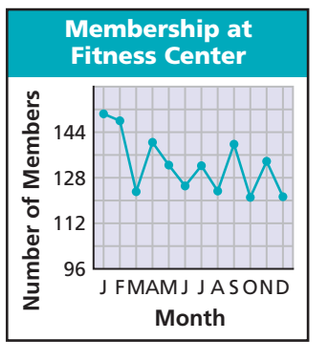
10-4 Lesson Quiz

Explain why each graph is misleading.

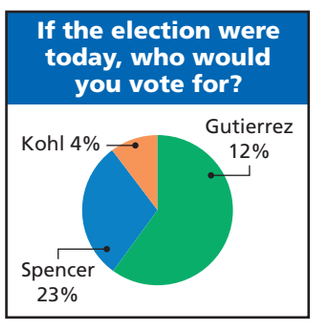
1.



2.



3.



4. A researcher asks 5 people if they like to snow ski. Four people respond yes. Explain why the following statement is misleading: "80% of people like to snow ski."
For answers, see Additional Answers.

Also available online

Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Examine different sampling methods and sources of bias.

Online Edition

Teach

Remember

Students review random samples.

Science In science classes, students learn about how Earth is divided into stratified layers. Explain that in a stratified random sample, the population is divided into groups, or layers.

Learn It Online
State Resources Online

Sampling and Bias

If you wanted to collect data about a very large group of people, you would most likely need to survey a smaller group. The large group that contains all the people you could survey is called a *population*. The smaller group is called a *sample*.

You have learned that a random sample is a good way of collecting data that is unbiased. There are different ways of selecting a random sample.

Random Samples		
Type	**Definition**	**Example**
Simple Random Sample	Every member of the population has an equal chance of being chosen for the sample.	The names of all students in your class are placed in a hat and three names are chosen at random.
Stratified Random Sample	The population is divided into similar groups. Then a simple random sample is chosen from each group.	Your class is divided into boys and girls and two students are chosen at random from each group.
Systematic Random Sample	A member of the population is chosen for the sample at a regular interval.	Every third student who comes into the classroom is chosen.

Example 1

In each situation, identify the population and the sample. Tell whether each sample is a simple, stratified, or systematic random sample.

A **For one week, the manager of a pet supplies store asks every tenth customer what brand of pet food they buy.**

population: all customers at the pet supplies store during one week

sample: every tenth customer during that same week

The sample is systematic because one member of the population is chosen for the sample at a regular interval.

B **Every person who enters a theater one evening places their ticket stub in a bowl. The theater owner chooses five ticket stubs to award prizes.**

population: all people who put their ticket stub in the bowl

sample: five ticket stubs chosen from the bowl

The sample is simple because every member of the population has the same chance of being chosen.

C **One student from each classroom at a school is chosen at random for a committee.**

population: all students at a school

sample: one student from each classroom

The sample is stratified because the population is divided into similar groups and one member is chosen at random from each group.

Try This

1. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.
2. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.

A random sample is not biased because no part of the population is favored over another. In a biased sample, one or more parts of the population have an advantage for being chosen for the sample.

There are two main types of biased samples.

Biased Samples		
Type	**Definition**	**Example**
Convenience Sample	Those members of the population that are easily accessed are chosen for the sample.	A reporter questions people he personally knows.
Voluntary Response Sample	Members of the population who want to participate make up the sample.	A reporter questions people who fill out comment cards and indicate that they would like to be contacted.

Example 2

In each situation, identify the population and the sample. Tell whether each sample is a convenience or voluntary response sample. Explain why the sample is biased.

A **A website asks visitors to complete a survey about Internet usage.**

population: all visitors to the website

sample: those visitors who choose to complete the survey

The sample is a voluntary response sample because visitors to the website chose whether to complete the survey. The sample is biased because those visitors who choose to complete the survey may not be representative of all visitors to the website.

B **A reporter asks people leaving a shopping center through one door about their shopping habits.**

population: all people at the shopping center

sample: people who leave from one door at the shopping center

The sample is a convenience sample because the people leaving through the chosen door are easily accessed. The sample is biased because people leaving through another door do not have an opportunity to be chosen.

Try This

3. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.
4. Assume that you want an unbiased sample. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.
5. Describe how a biased sample could be chosen for this same situation.

Close

Assess

After students have described how they would choose a random sample for their given topic, discuss how sources of bias could be avoided.

Answers to *Try This*

1–2. Check students' work.

3–5. Check students' work.

Connecting Algebra to Data Analysis **723**

SECTION 10A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

CHAPTER 10

SECTION 10A

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Data Analysis

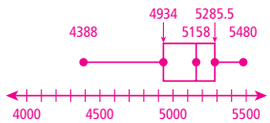
USA! USA! USA! In 708 B.C.E., the Olympic Games included the pentathlon. This event included the discus, javelin, long jump, running, and wrestling. The winner of the pentathlon was considered a "super athlete." The modern pentathlon of the Olympic Games includes five disciplines: target shooting, fencing, swimming, horseback riding, and cross-country running.

1. The pentathlon includes five events. What type of graph should be used to show what percent of this event is made up of each discipline? **circle graph**
2. Do you think the winner of the modern pentathlon must win each of the five events? Explain.
3. Thirty-two men competed in the modern pentathlon in the 2004 Olympic Games. The chart shows the total points for the competitors.

Pentathlon Scores						
5480	5428	5392	5356	5340		
5332	5320	5295	5276	5252	5200	5196
5192	5184	5180	5172	5144	5132	5084
5084	5068	5016	4976	4936	4932	4916
4904	4848	4732	4676	4420	4388	

Find the mean, median, and mode of this data set. Which value best describes the data? Explain.

4. Find the minimum, first quartile, third quartile, and maximum values of the data set above. Use these values and the median to make a box-and-whisker plot of the data. **min.: 4388; Q1: 4934; Q3: 5285.5; max.: 5480**



2. No; an athlete could score very well in 1 event but have low scores in the other 4 events. Another athlete with 4 high scores and 1 low score would have a better total score.

3. Mean: 5089.09375; median: 5158; mode: 5084; the mean best describes the data because there are no outliers.

(all), Stuart Franklin/Getty Images

INTERVENTION

Scaffolding Questions

1. What type of graph compares the parts to the whole? circle graph
2. What if an athlete wins one event but has low scores in the other four events? Another athlete might win four events but have a low score in one event. This athlete could have the highest total score.
3. Are there any outliers? yes How does an outlier affect the mean? A very small outlier makes the mean smaller, and a very large outlier makes the mean greater.
4. How do you show an outlier on a box-and-whisker plot? Draw a dot and do not connect it to the rest of the graph.

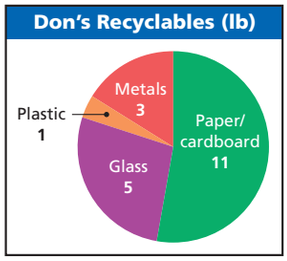
Extension

How would you display the data if you wanted to show the number of athletes who had total scores within a certain interval? a frequency table or a histogram

Quiz for Lessons 10-1 Through 10-4

10-1 Organizing and Displaying Data

Use the circle graph for 1–3.



1. Which material represents over 50% of Don's recyclables? paper/cardboard
2. How many pounds of materials does Don recycle? 20
3. What percent of Don's recyclables are glass? 25%
4. The table shows the total proceeds for a fund-raiser at various times during the day. Choose a type of graph to display the given data. Make the graph, and explain why you chose that type of graph.

Time	Total Proceeds (thousand $)	Time	Total Proceeds (thousand $)
3:00 P.M.	1.5	6:00 P.M.	6.5
4:00 P.M.	2	7:00 P.M.	8
5:00 P.M.	4	8:00 P.M.	9.5

10-2 Frequency and Histograms

5. The Miller family is going to buy a new car. The approximate prices of the cars they are considering are given. Use the data to make a stem-and-leaf plot.

   $24,000, $28,000, $26,000, $32,000, $30,000, $41,000, $27,000, $22,000, $26,000, $33,000

6. The number of people at a caterer's last 12 parties are given below.

   16, 18, 17, 19, 15, 25, 18, 17, 18, 16, 17, 19

   a. Use the data to make a frequency table with intervals.

   b. Use your frequency table from part **a** to make a histogram.

10-3 Data Distributions

7. The daily high temperatures on 14 consecutive days in one city were 59 °F, 49 °F, 48 °F, 46 °F, 47 °F, 51 °F, 49 °F, 43 °F, 45 °F, 52 °F, 51 °F, 51 °F, 51 °F, and 38 °F.

   a. Find the mean, median, and mode of the temperatures. mean: 48.57; med.: 49; mode: 51

   b. Which value describes the average high temperature for the 14 days? mean

   c. Which value best describes the high temperatures? Explain.

8. Use the temperature data above to make a box-and-whisker plot.

10-4 Misleading Graphs and Statistics

9. The graph shows the value of a company's stock over time. Explain why the graph is misleading. What might people be influenced to believe because of the graph? Who might want to use this graph?
10. The results of an online survey of 230 people showed that 92% of the population felt very comfortable using technology. Explain why this statistic is misleading.

READY TO GO ON?

SECTION 10A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

4–6, 7c–10. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 10A: Worksheets	*Ready to Go On? Intervention*, Section 10A: Online	
Lesson 10-1	10-1 Intervention	Activity 10-1	Diagnose and Prescribe Online
Lesson 10-2	10-2 Intervention	Activity 10-2	
Lesson 10-3	10-3 Intervention	Activity 10-3	
Lesson 10-4	10-4 Intervention	Activity 10-4	

YES ENRICH

***Ready to Go On? Enrichment*, Section 10A**
Worksheets
Online

SECTION
10B Probability

One-Minute Section Planner

Lesson	Lab Resources	Materials
Algebra Lab Simulations • Use a number cube to simulate a probability experiment. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** number cube (MK)
Lesson 10-5 Experimental Probability • Determine the experimental probability of an event. • Use experimental probability to make predictions. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** number cube (MK)
Technology Lab Use Random Numbers • Use a graphing calculator to find experimental probabilities by generating random numbers. ☐ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 10-6 Theoretical Probability • Determine the theoretical probability of an event. • Convert between probabilities and odds. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** number cube (MK), spinner (MK)
Lesson 10-7 Independent and Dependent Events • Find the probability of independent events. • Find the probability of dependent events. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** red and black marbles, checkers, playing cards
Algebra Lab Compound Events • Find the probability of compound events. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 5: 3, 5, 7, 9, 11a, 11b, 13, 15, 17, 19, 21a, 21b, 27, 29, 31, 33, 35, 37a, 37b

Lesson 6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 31, 35, 37

Lesson 7: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23a, 23b, 25, 27, 29a, 29b, 29c, 29d, 31, 35, 37, 39, 41, 43

Lesson 8: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25a, 25b, 27, 29, 31, 33a, 33b, 37, 39, 41

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

PROBABILITY

Lessons 10-5 to 10-6

A *probability* is a number between 0 and 1 inclusive that measures the likelihood of an event. When an event consists of finitely many equally likely outcomes, the *theoretical probability* of the event is the ratio of the number of outcomes in the event to the total number of possible outcomes. The probability of an impossible event is 0; the probability of a certain event is 1. When the set of all possible outcomes is finite, the converses of these statements are also true.

Probabilities may also be found by experimental means. The *experimental probability* of an event is the ratio of the number of times the event occurs to the total number of trials. For example, if you toss a coin 100 times and it comes up heads 54 times, the experimental probability of heads is $\frac{54}{100}$, or 0.54.

The *Law of Large Numbers* states that the experimental probability of an event approaches the theoretical probability as the number of trials becomes arbitrarily large. In the above examples, as the number of coin tosses increases, the experimental probability of heads will tend to approach the theoretical probability, 0.5.

PROBABILITY OF MULTIPLE EVENTS

Lesson 10-7

Two events are *independent* if the occurrence of one does not affect the probability of the other. If A and B are independent, then $P(A \text{ and } B)$, also written $P(A \cap B)$, is $P(A) \cdot P(B)$. More generally, events A_1, A_2, …, A_n are *mutually independent* if the occurrence of any one does not affect the probability of any other. Then, $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_n)$.

Mutually exclusive events cannot occur in the same trial. For example, a coin cannot simultaneously show heads and tails. If A and B are mutually exclusive, then $P(A \text{ or } B)$, also written $P(A \cup B)$, is $P(A) + P(B)$. This addition rule is generally used when finding probability for a single trial, while the multiplication rule for independent events is usually applied to two or more separate trials, performed either simultaneously or in succession.

Organizer

Use with the lesson *Experimental Probability*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a number cube to simulate a probability experiment.

Materials: number cube

Online Edition
TechKeys, Graphing Calculator

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

To *simulate* means to pretend to experience something. It is a word also used outside of mathematics.

A number cube (MK) is only one way to create the simulation in the activity. A spinner with six equal sections would also work.

Close

Key Concept

A simulation can replace the need for actually performing an experiment. This is particularly helpful if the experiment would be difficult or inconvenient to perform.

Assessment

Journal Have students explain when and how a simulation is used.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

Learn It Online
State Resources Online

10-5 Algebra LAB

Use with Experimental Probability

Simulations

A simulation can be used to model an experiment that would be difficult or inconvenient to actually perform. In this lab, you will conduct simulations.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

Activity

The local movie theater is offering an opportunity for customers to win a free night at the movies. To win, you must collect six different letters to spell CINEMA. Each movie ticket sold during this promotion will have one of the six letters stamped on the back of the ticket. An equal number of tickets will be stamped with each of the letters.

1. Since there are six different letters that appear on the tickets an equal number of times, you can use a number cube to simulate collecting the six letters.

   Each of the numbers on the number cube will represent a letter. Each roll of the number cube will represent purchasing one movie ticket, and the number rolled will represent the letter stamped on the ticket.

2. The table shows the results of rolling the number cube until each number has been rolled once.
   a. Based on the results shown in the table, how many rolls did it take to get all six numbers?
   b. Based on the results in the table, how many movie tickets would you expect to have to buy to get all six letters? If you purchased this number of tickets, would you be sure to win? Explain.

Number on Cube	Letter on Ticket	Frequency
1	C	|
2	I	𝍸 |
3	N	||||
4	E	||
5	M	||
6	A	|||

Try This

1. Repeat the simulation four more times and record the results. **Check students' work.**
2. Find the average number of rolls from all five simulations $\left(\frac{\text{total number of rolls from 5 simulations}}{5}\right)$. **Possible answer:** $\frac{18 + 15 + 9 + 12 + 27}{5} = 16.2$
3. Based on your answer to Problem 2, how many movie tickets would you expect to have to buy to get all six letters? Is this number different from the answer you gave based on the results in the table above? **Check students' work.**
4. Would any of your answers have been different if you had used a different correspondence between the numbers and letters? Explain. **No; if 1 number corresponds to 1 letter, the results should be about the same.**

Sam Dudgeon/HMH

Answers to Activity

2a. 18 rolls

b. No; a simulation is just a model and there may be different results each time the simulation is performed.

COMMON CORE

10-5 Experimental Probability

CC.9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Objectives
Determine the experimental probability of an event.

Use experimental probability to make predictions.

Vocabulary
experiment
trial
outcome
sample space
event
probability
experimental probability
prediction

Why learn this?
Experimental probability can be used by manufacturers for quality control. (See Example 4.)

An **experiment** is an activity involving chance. Each repetition or observation of an experiment is a **trial**, and each possible result is an **outcome**. The **sample space** of an experiment is the set of all possible outcomes.

Experiment	Rolling a number cube	Tossing a coin	Spinning a game spinner
Sample Space	{1, 2, 3, 4, 5, 6}	{heads, tails}	{red, blue, green, yellow}

EXAMPLE 1 **Identifying Sample Spaces and Outcomes**

Identify the sample space and the outcome shown for each experiment.

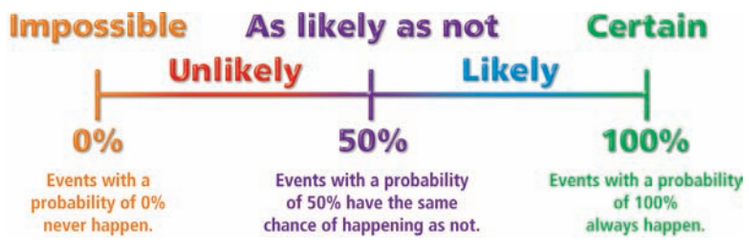
A **tossing two coins**
Sample space: {HH, HT, TH, TT}
Outcome shown: heads, tails (H, T)

B **spinning a game spinner**
Sample space: {yellow, red, blue, green}
Outcome shown: green

CHECK IT OUT! **1.** Identify the sample space and the outcome shown for the experiment: rolling a number cube.

sample space: {1, 2, 3, 4, 5, 6}; outcome shown: 3

An **event** is an outcome or set of outcomes in an experiment. **Probability** is the measure of how likely an event is to occur. Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



(tl), (tr), (bc) Sam Dudgeon/HMH; (tc), (tc), United States coin images from the United States Mint

10-5 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Determine the experimental probability of an event.

Use experimental probability to make predictions.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Write the equivalent percent.

1. $\frac{3}{10}$ 30% **2.** $\frac{1}{4}$ 25%

3. $\frac{18}{90}$ 20% **4.** $\frac{88}{550}$ 16%

Write the equivalent fraction.

5. 50% $\frac{1}{2}$ **6.** 10% $\frac{1}{10}$

7. 40% $\frac{2}{5}$ **8.** 35% $\frac{7}{20}$

Write the equivalent decimal.

9. 18% 0.18 **10.** 50% 0.5

11. $\frac{1}{5}$ 0.2 **12.** $\frac{9}{10}$ 0.9

Also available online

Math Fact !?!

South African mathematician John Kerrich tossed a coin 10,000 times. The coin landed with heads showing 5067 times.

COMMON CORE

CC.9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Experimental Probability

You can use a graphing calculator to simulate tossing a coin. In this simulation, a result of 0 represents tails, and a result of 1 represents heads.

1. Press MATH and use the arrow keys to move to the **PRB** menu. Then choose the random integer function by selecting **5:randInt(.**

2. Press 0 , 1 to give the range of integers. Press ENTER to simulate tossing a coin.

3. You can press ENTER repeatedly to simulate multiple coin tosses.
4. Simulate tossing a coin 10 times. How many of the tosses resulted in heads? Possible answer: 6
5. Combine your results with those of other students. What is the total number of coin tosses? How many of the tosses resulted in heads? Possible answer: 50 total coin tosses; 27 tosses resulted in heads.

THINK AND DISCUSS

6. **Explain** how many times you would expect the result to be heads if you tossed a coin 1000 times. about 500 times

Motivate

Roll a number cube (MK) several times and record the results on the board. Tell students that you have just performed a probability experiment. Tell students they will learn about experimental probability, which is used to make predictions about the stock market, sports, and the behavior of humans and animals.

Explorations and answers are provided in your online resources.

Power Presentations with PowerPoint®

Additional Examples

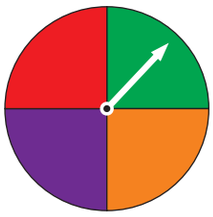
Example 1

Identify the sample space and the outcome shown for each experiment.

A. rolling a number cube

sample space: {1, 2, 3, 4, 5, 6}; outcome shown: 4

B. spinning a spinner



sample space: {red, green, purple, orange}; outcome shown: green

Example 2

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

A. A shoe selected from a pair of shoes fits the right foot. as likely as not

B. Katrina correctly guesses the last digit of a phone number. unlikely

C. Max pulls a green marble from a bag of all green marbles. certain

D. A randomly selected month contains the letter R. likely

Example 3

An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Green	15
Orange	10
Purple	8
Pink	7

A. The spinner lands on orange. $\frac{1}{4}$

B. The spinner does not land on green. $\frac{5}{8}$

Also available online

EXAMPLE 2 **Estimating the Likelihood of an Event**

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

A **There are 31 days in August.**
August always has 31 days. This event is *certain.*

B **Carlos correctly guesses a number between 1 and 1000.**
Carlos must pick one outcome out of 1000 possible outcomes. This event is *unlikely.*

C **A coin lands heads up.**
Heads is one of two possible outcomes. This event is *as likely as not.*

D **Cecilia rolls a 10 on a standard number cube.**
A standard number cube is numbered 1 through 6. This event is *impossible.*

CHECK IT OUT! **2.** Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe the event: Anthony rolls a number less than 7 on a standard number cube. **certain**

You can estimate the probability of an event by performing an experiment. The **experimental probability** of an event is the ratio of the number of times the event occurs to the number of trials. The more trials performed, the more accurate the estimate is likely to be.

Know it! Note

Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

EXAMPLE 3 **Finding Experimental Probability**

An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	7
Blue	8
Green	5

Helpful Hint

Probabilities can be expressed as fractions, decimals, or percents. For Example 3, the probabilities are:
3A: $\frac{2}{5} = 0.4 = 40\%$
3B: $\frac{3}{4} = 0.75 = 75\%$

A **spinner lands on blue**

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{8}{7+8+5} = \frac{8}{20} = \frac{2}{5}$$

B **spinner does not land on green**

When the spinner does not land on green, it must land on red or blue.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{7+8}{7+8+5} = \frac{15}{20} = \frac{3}{4}$$

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLES 1–2

- If an event is impossible, what is the relationship between the event and the sample space? What if the event is certain?

EXAMPLE 3

- How do you find the experimental probability of an event not occurring?

2 Teach

Guided Instruction

Introduce an experiment as a process that leads to outcomes. Practice listing the sample spaces for several experiments. Refer to the continuum on the first page of this lesson when working through **Example 2.** Mention that many people say *fifty-fifty* instead of *as likely as not.*

Teaching Tip **Cognitive Strategies** Reinforce definitions by using the vocabulary in the lesson when discussing the lesson.

CHECK IT OUT! **Use the information in Example 3 to find the experimental probability of each event.**

3a. spinner lands on red $\frac{7}{20}$

3b. spinner does not land on red $\frac{13}{20}$

You can use experimental probability to make *predictions*. A **prediction** is an estimate or guess about something that has not yet happened.

EXAMPLE 4 *Quality Control Application*

A manufacturer inspects 800 light bulbs and finds that 796 of them have no defects.

A **What is the experimental probability that a light bulb chosen at random has no defects?**

Find the experimental probability that a light bulb has no defects.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{796}{800}$$

$$= 99.5\%$$

The experimental probability that a light bulb has no defects is 99.5%.

B **The manufacturer sent a shipment of 2400 light bulbs to a retail store. Predict the number of light bulbs in the shipment that are likely to have no defects.**

Find 99.5% of 2400.

$0.995(2400) = 2388$

The manufacturer predicts that 2388 light bulbs have no defects.

CHECK IT OUT! **4.** A manufacturer inspects 1500 electric toothbrush motors and finds that 1497 of them have no defects.

a. What is the experimental probability that a motor chosen at random will have no defects? **99.8%**

b. There are 35,000 motors in a warehouse. Predict the number of motors that are likely to have no defects. **34,930**

MATHEMATICAL PRACTICES

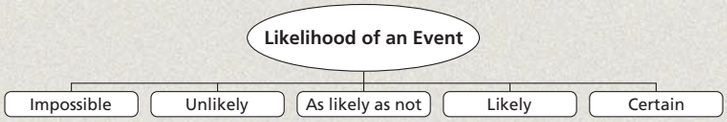
THINK AND DISCUSS

1. Explain the difference between an outcome and an event.

2. Is the experimental probability of an event always the same? Explain why or why not.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an event that has the given likelihood.



COMMON ERROR ALERT

Students might list extremely unlikely events as impossible and extremely likely events as certain. Remind students that if an event is impossible, then there is absolutely no way it could ever happen. If an event is certain, there is absolutely no way it could not happen.

Power Presentations with PowerPoint®

Additional Examples

Example 4

A manufacturer inspects 500 strollers and finds that 498 have no defects.

a. What is the experimental probability that a stroller chosen at random has no defects? 99.6%

b. The manufacturer shipped 3500 strollers to a distribution center. Predict the number of strollers that are likely to have no defects. 3486

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- How do you use experimental probability to make a prediction?

Teaching Tip **Multiple Representations** Students can use proportions in part **b** of **Example 4:**

$$\frac{\text{no. w/o defects}}{\text{no. inspected}} = \frac{x}{\text{no. in shipment}}.$$

3 Close

Summarize

Tell students that to determine the experimental probability of an event, they need to count the number of trials in the experiment. Then they should count the number of times the event occurred and divide by the number of trials. Predictions can be made by using experimental probabilities.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. An outcome is 1 possible result of an experiment. An event is a set of outcomes.
2. No; different outcomes may occur with different frequencies each time the experiment is carried out. The more trials performed, the more accurate the estimate is likely to be.
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 12–17, 23, 26, 27
- **Average** 12–17, 23, 26, 27
- **Advanced** 12–16 even, 26, 27, 34–37

If you finished Examples **1–4**
- **Basic** 12–23, 25–32
- **Average** 12–22, 24–36
- **Advanced** 12–20 even, 21, 22, 24–37

Homework Quick Check
Quickly check key concepts.
Exercises: 12, 16, 20, 21, 26, 28

Teaching Tip **Inclusion** Some students may benefit from noting that the answer to **Exercise 8** plus the answer to **Exercise 10** equals 1.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 11, 21–23, 28, 32, 37

Reason abstractly and quantitatively. Exercise 26

Construct viable arguments and critique the reasoning of others. Exercises 24–25

Attend to precision. Exercise 27

Learn It Online
State Resources Online

10-5 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Give an example of an *event* that has two possible *outcomes*.
Possible answer: picking a shoe at random from a pair of shoes

SEE EXAMPLE 1

Identify the sample space and the outcome shown for each experiment.

2. rolling a number cube

2. sample space: {1, 2, 3, 4, 5, 6}; outcome shown: 2

3. spinning a spinner

sample space: {blue, red, yellow, green}; outcome shown: red

4. tossing 3 coins

sample space: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}; outcome shown: THT

SEE EXAMPLE 2

Write *impossible, unlikely, as likely as not, likely*, or *certain* to describe each event.

5. Peter was born in January. Thomas was born in June. Peter and Thomas have the same birthday. impossible

6. The football team won 9 of its last 10 games. The team will win the next game. likely

7. A board game has a rule that if you roll the game cube and get a 6, you get an extra turn. You get an extra turn on your first roll. unlikely

SEE EXAMPLE 3

An experiment consists of rolling a number cube. Use the results in the table to find the experimental probability of each event.

Outcome	1	2	3	4	5	6
Frequency	5	6	2	2	3	7

8. rolling a 6 $\frac{7}{25}$

9. rolling an even number $\frac{3}{5}$

10. not rolling a 6 $\frac{18}{25}$

SEE EXAMPLE 4

11. **Sports** One game of bowling consists of ten frames. Elyse usually rolls 3 strikes in each game.
   a. What is the experimental probability that Elyse will roll a strike on any frame? 30%
   b. Predict the number of strikes Elyse will roll in 18 games. 54

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–17	2
18–20	3
21	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Identify the sample space and the outcome shown for each experiment.

12. tossing two coins

sample space: {HH, HT, TH, TT}; outcome shown: TT

13. spinning a spinner

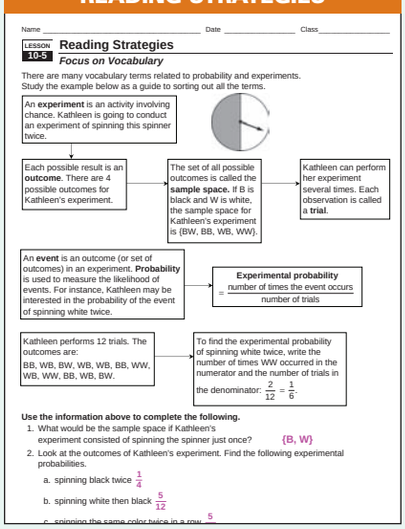
sample space: {blue, red, yellow}; outcome shown: blue

14. selecting a marble

14. sample space: {blue, yellow, green, red}; outcome shown: green

(tl), (tc), (bc), (br) Sam Dudgeon/HMH; (tr), (bl), PhotoDisc/Getty Images

READING STRATEGIES



Name _____ Date _____ Class _____

LESSON 10-5 **Reading Strategies**
Focus on Vocabulary

There are many vocabulary terms related to probability and experiments. Study the example below as a guide to sorting out all the terms.

An **experiment** is an activity involving chance. Kathleen is going to conduct an experiment of spinning this spinner twice.

Each possible result is an **outcome**. There are 4 possible outcomes for Kathleen's experiment.

The set of all possible outcomes is called the **sample space.** If B is black and W is white, the sample space for Kathleen's experiment is {BW, BB, WB, WW}.

Kathleen can perform her experiment several times. Each observation is called a **trial.**

An **event** is an outcome (or set of outcomes) in an experiment. **Probability** is used to measure the likelihood of events. For instance, Kathleen may be interested in the probability of the event of spinning white twice.

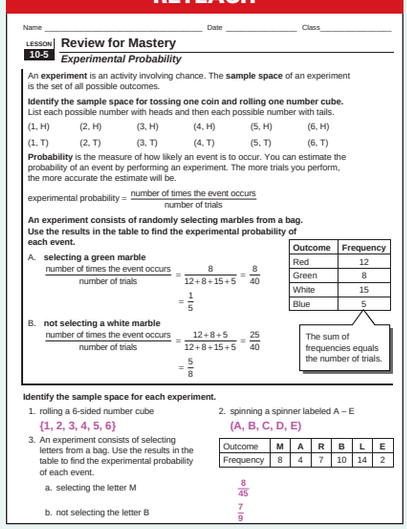
Experimental probability = number of times the event occurs / number of trials

Kathleen performs 12 trials. The outcomes are: BB, WB, BW, WB, WB, BB, WW, WB, WW, BB, WB, BW.

To find the experimental probability of spinning white twice, write the number of times WW occurred in the numerator and the number of trials in the denominator: $\frac{2}{12} = \frac{1}{6}$.

Use the information above to complete the following.
1. What would be the sample space if Kathleen's experiment consisted of spinning the spinner just once? {B, W}
2. Look at the outcomes of Kathleen's experiment. Find the following experimental probabilities.
   a. spinning black twice $\frac{1}{4}$
   b. spinning white then black $\frac{5}{12}$
   c. spinning the same color twice in a row $\frac{5}{}$

RETEACH



Name _____ Date _____ Class _____

LESSON 10-5 **Review for Mastery**
Experimental Probability

An **experiment** is an activity involving chance. The **sample space** of an experiment is the set of all possible outcomes.

Identify the sample space for tossing one coin and rolling one number cube.
List each possible number with heads and then each possible number with tails.

(1, H) (2, H) (3, H) (4, H) (5, H) (6, H)
(1, T) (2, T) (3, T) (4, T) (5, T) (6, T)

Probability is the measure of how likely an event is to occur. You can estimate the probability of an event by performing an experiment. The more trials you perform, the more accurate the estimate will be.

experimental probability = number of times the event occurs / number of trials

An experiment consists of randomly selecting marbles from a bag. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	12
Green	8
White	15
Blue	5

A. **selecting a green marble**
$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{8}{12+8+15+5} = \frac{8}{40} = \frac{1}{5}$

B. **not selecting a white marble**
$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{12+8+5}{12+8+15+5} = \frac{25}{40} = \frac{5}{8}$

The sum of frequencies equals the number of trials.

Identify the sample space for each experiment.
1. rolling a 6-sided number cube {1, 2, 3, 4, 5, 6}
2. spinning a spinner labeled A – E {A, B, C, D, E}
3. An experiment consists of selecting letters from a bag. Use the results in the table to find the experimental probability of each event.

Outcome	M	A	R	B	L	E
Frequency	8	4	7	10	14	2

   a. selecting the letter M $\frac{8}{45}$
   b. not selecting the letter B $\frac{7}{9}$

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

15. Marlo purchased a new pair of shoes. She takes one shoe out of the box. The shoe is for the left foot. **as likely as not**

16. Sam takes the bus to school. The bus came late twice in the last two weeks. The bus will be late today. **unlikely**

17. Tammy dropped two quarters on the floor. At least one of them lands heads up. **likely**

An experiment consists of randomly choosing a marble from a bag. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	4
Blue	6
Green	6
Yellow	9

18. choosing a yellow marble $\frac{9}{25}$

19. choosing a blue marble $\frac{6}{25}$

20. not choosing a green marble $\frac{19}{25}$

21. **Sports** A ski lodge inspects 80 skis and finds 4 to be defective.
    a. What is the experimental probability that a ski chosen at random will be defective? **5%**
    b. The lodge has 420 skis. Predict the number of skis that are likely to be defective. **21**

22. The table shows the results of a survey asking students the season of their birthday. What is the experimental probability that a student has a birthday during the summer? $\frac{17}{50}$

Season	Birthdays
Fall	39
Winter	27
Spring	33
Summer	51

23. You and your friend can either go swimming or to a movie on Thursday. The weather forecast says there is a 70% chance of rain on Thursday. Should you plan on going swimming or to a movie? Explain.

23. A 70% chance of rain means that it is likely to rain on Thu., so you should plan to go to a movie.

24. **Critical Thinking** Tell why it is important to repeat an experiment many times.

24. An unlikely event may occur in any single experiment. Repeating the experiment many times gives a more accurate picture of exp. probabilities.

25. **Write About It** Explain what it means for an event to have a 50-50 chance of happening. **The event is as likely to happen as not.**

26. How many outcomes are in the sample space for an experiment consisting of rolling two standard number cubes? **36**

27. **Estimation** A manufacturing company produced 986 units in one day. Of those, 9 units were found to be defective. Estimate the experimental probability that a unit produced that day was defective. Then predict approximately how many units will be defective when 5680 units are produced in one week. **about 1%; about 57**

MULTI-STEP TEST PREP

28. In a standard deck of cards, there are 13 cards in each of four suits: hearts, diamonds, clubs, and spades. The hearts and diamonds are red and the clubs and spades are black. Ricardo randomly drew cards from a standard deck of 52 cards. The table shows the results. $\frac{2}{7}$

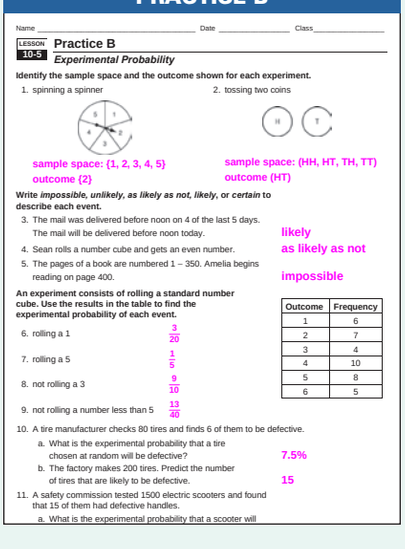
Outcome	Frequency
Hearts	7
Diamonds	7
Clubs	8
Spades	6

    a. Find the experimental probability of drawing a club.
    b. Find the experimental probability of drawing a black suit. $\frac{1}{2}$

Sam Dudgeon/HMH

MULTI-STEP TEST PREP **Exercise 28** involves finding the probability of an event involving a standard deck of cards. This exercise prepares students for the Multi-Step Test Prep.

PRACTICE A
PRACTICE C
PRACTICE B



Name _____ Date _____ Class _____

LESSON 10-5 **Practice B**
Experimental Probability

Identify the sample space and the outcome shown for each experiment.

1. spinning a spinner
   sample space: {1, 2, 3, 4, 5}
   outcome {2}
2. tossing two coins
   sample space: {HH, HT, TH, TT}
   outcome (HT)

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

3. The mail was delivered before noon on 4 of the last 5 days. The mail will be delivered before noon today. **likely**
4. Sean rolls a number cube and gets an even number. **as likely as not**
5. The pages of a book are numbered 1 – 350. Amelia begins reading on page 400. **impossible**

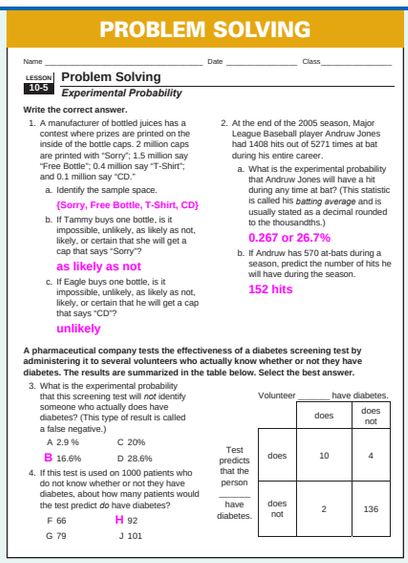
An experiment consists of rolling a standard number cube. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
1	6
2	7
3	4
4	10
5	8
6	5

6. rolling a 1 $\frac{3}{20}$
7. rolling a 5 $\frac{1}{5}$
8. not rolling a 3 $\frac{9}{10}$
9. not rolling a number less than 5 $\frac{13}{40}$

10. A tire manufacturer checks 80 tires and finds 6 of them to be defective.
    a. What is the experimental probability that a tire chosen at random will be defective? **7.5%**
    b. The factory makes 200 tires. Predict the number of tires that are likely to be defective. **15**
11. A safety commission tested 1500 electric scooters and found that 15 of them had defective handles.
    a. What is the experimental probability that a scooter will

PROBLEM SOLVING



Name _____ Date _____ Class _____

LESSON 10-5 **Problem Solving**
Experimental Probability

Write the correct answer.

1. A manufacturer of bottled juices has a contest where prizes are printed on the inside of the bottle caps. 2 million caps are printed with "Sorry"; 1.5 million say "Free Bottle"; 0.4 million say "T-Shirt"; and 0.1 million say "CD."
   a. Identify the sample space. **{Sorry, Free Bottle, T-Shirt, CD}**
   b. If Tammy buys one bottle, is it impossible, unlikely, as likely as not, likely, or certain that she will get a cap that says "Sorry"? **as likely as not**
   c. If Eagle buys one bottle, is it impossible, unlikely, as likely as not, likely, or certain that he will get a cap that says "CD"? **unlikely**
2. At the end of the 2005 season, Major League Baseball player Andruw Jones had 1408 hits out of 5271 times at bat during his entire career.
   a. What is the experimental probability that Andruw Jones will have a hit during any time at bat? (This statistic is called his *batting average* and is usually stated as a decimal rounded to the thousandths.) **0.267 or 26.7%**
   b. If Andruw has 570 at-bats during a season, predict the number of hits he will have during the season. **152 hits**

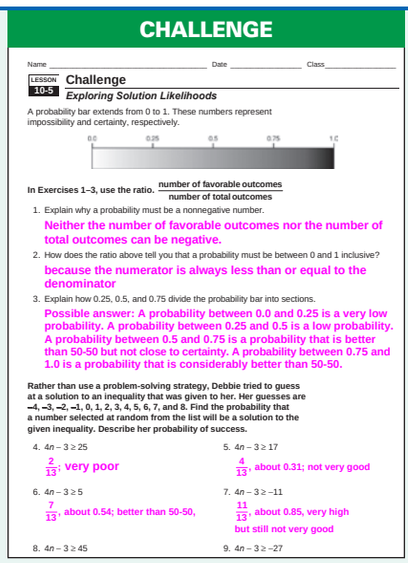
A pharmaceutical company tests the effectiveness of a diabetes screening test by administering it to several volunteers who actually know whether or not they have diabetes. The results are summarized in the table below. Select the best answer.

3. What is the experimental probability that this screening test will *not* identify someone who actually does have diabetes? (This type of result is called a false negative.)
   A 2.9 % C 20%
   B 16.6% D 28.6%
4. If this test is used on 1000 patients who do not know whether or not they have diabetes, about how many patients would the test predict *do* have diabetes?
   F 66 **H** 92
   G 79 J 101

Volunteer _____ have diabetes.

Test predicts that the person _____ have diabetes.	does	does not
does	10	4
does not	2	136

CHALLENGE



Name _____ Date _____ Class _____

LESSON 10-5 **Challenge**
Exploring Solution Likelihoods

A probability bar extends from 0 to 1. These numbers represent impossibility and certainty, respectively.

0.0 0.25 0.5 0.75 1.0

In Exercises 1–3, use the ratio. $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$

1. Explain why a probability must be a nonnegative number. **Neither the number of favorable outcomes nor the number of total outcomes can be negative.**
2. How does the ratio above tell you that a probability must be between 0 and 1 inclusive? **because the numerator is always less than or equal to the denominator**
3. Explain how 0.25, 0.5, and 0.75 divide the probability bar into sections. **Possible answer: A probability between 0.0 and 0.25 is a very low probability. A probability between 0.25 and 0.5 is a low probability. A probability between 0.5 and 0.75 is a probability that is better than 50-50 but not close to certainty. A probability between 0.75 and 1.0 is a probability that is considerably better than 50-50.**

Rather than use a problem-solving strategy, Debbie tried to guess at a solution to an inequality that was given to her. Her guesses are –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, and 8. Find the probability that a number selected at random from the list will be a solution to the given inequality. Describe her probability of success.

4. $4n - 3 \geq 25$
   $\frac{2}{13}$; **very poor**
5. $4n - 3 \geq 17$
   $\frac{4}{13}$; **about 0.31; not very good**
6. $4n - 3 \geq 5$
   $\frac{7}{13}$; **about 0.54; better than 50-50,**
7. $4n - 3 \geq -11$
   $\frac{11}{13}$; **about 0.85, very high but still not very good**
8. $4n - 3 \geq 45$
9. $4n - 3 \geq -27$

TEST PREP DOCTOR Students who chose **C** for **Exercise 29** may have been considering the probability of rolling a number less than 4 on only one number cube.

If students need help starting **Exercise 32,** have them write the number of tails as x and the number of heads as $x + 6$.

Journal

Have students explain how to use experimental probability to make a prediction.

ALTERNATIVE ASSESSMENT

Have students roll a number cube 20 times and record their results in a table as in **Example 3.** Then have them use their results to find the experimental probability of rolling a 5 and use it to make a prediction about how many times they would roll a 5 in 100 trials.

Power Presentations with PowerPoint®

10-5 Lesson Quiz

1. Identify the sample space and the outcome shown for selecting a marble.

sample space: {red, green, yellow}; outcome shown: red

2. An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of landing on blue. $\frac{3}{10}$

Outcome	Frequency
Red	9
Blue	6
Yellow	5

3. The neighbors' dog barked at Tana the last 4 out of 5 times she walked by their house.

a. What is the experimental probability that the dog barks at Tana when she walks past the house? 80%

b. Predict the number of times the dog will bark at Tana if she walks past the house 45 times. 36

Also available online

TEST PREP

29. Alex rolls two standard number cubes. What is the likelihood that the sum of the numbers is less than 4?

Ⓐ Impossible Ⓑ Unlikely Ⓒ As likely as not Ⓓ Likely

30. A community reported that $\frac{2}{3}$ of the residents had a pet and $\frac{1}{2}$ of the pet owners had a dog. If there are 84 residents in the community, how many residents are likely to have a dog?

Ⓕ 14 Ⓖ 28 Ⓗ 42 Ⓙ 56

31. What is the probability that a number chosen at random from the list below will be a solution of the inequality $3x + 2 \leq 23$?

$-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Ⓐ $\frac{17}{18}$ Ⓑ $\frac{8}{9}$ Ⓒ $\frac{1}{9}$ Ⓓ $\frac{1}{18}$

32. Short Response A coin was tossed 50 times. It landed showing heads 6 more times than it landed showing tails. What is the experimental probability of the coin landing on heads? Show your work. $\frac{14}{25}$; check students' work.

CHALLENGE AND EXTEND

A coin is tossed 3 times. Use the sample space for this experiment to describe each event below as *impossible, unlikely, as likely as not, likely,* or *certain.* Justify each answer.

33. At least 2 heads **34.** 2 heads and 1 tail

35. 2 tails and 1 head **36.** 3 tails

37. One coin is tossed 20 times.

a. The experimental probability of the coin showing heads is 65%. How many times did the coin show tails? 7

b. If the coin is tossed ten more times, how many more times must the coin land showing tails for the experimental probability of tails to be 50%? 8

35. Unlikely; there are 3 outcomes in this event (TTH, THT, HTT) among 8 outcomes in the sample space.

36. Unlikely; there is 1 outcome in this event (TTT) among 8 outcomes in the sample space.

Answers

33. As likely as not; there are 4 outcomes in this event (HHH, HHT, HTH, THH) among 8 outcomes in the sample space.

34. Unlikely; there are 3 outcomes in this event (HHT, HTH, THH) among 8 outcomes in the sample space.

10-5 Technology LAB

Use Random Numbers

Use with Experimental Probability

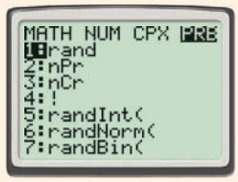
A calculator can be used to model an experiment that would be difficult or inconvenient to perform. To do this, you will use random numbers.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.S.IC.1 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Activity

You can use a calculator to explore the experimental probability that at least 2 people in a group of 6 people were born in the same month. Assume that all months are equally likely to be a person's birth month.



1. Represent each month with an integer. Since there are 12 months, use the numbers 1–12.

   To set your calculator up to generate random numbers, press MATH. Then use the arrow keys to highlight **PRB**. Select **5: randInt(**.

2. Now give the start number, 1, press , , and give the end number, 12.

   Each time you press ENTER the calculator will return an integer from 1 to 12.

3. You are considering a group of 6 people. This means you need 6 random numbers.

	Person					
	1	2	3	4	5	6
Trial 1	11	4	9	3	2	9
Trial 2	3	8	10	12	6	2

In the first trial, the number 9 appears twice. This means that two people have a birth day in the ninth month, September.

In the second trial, no number appears more than once. This means that none of the people were born in the same month.

Try This

1. Repeat the experiment until you have 10 trials of the experiment. Count the number of trials in which a number appears more than once. Divide this number by the number of trials, 10, to find the experimental probability that at least 2 people in a group of 6 people will have the same birth month. **Check students' work.**
2. Gather the results from at least 100 trials of the experiment. (Either perform all of the trials yourself or combine data with your classmates.) Using your results, what is the experimental probability that at least 2 people in a group of 6 people will have the same birth month? Compare the results from 100 trials to the results of 10 trials. **Check students' work; as the number of trials increases, the result should approach 78%.**
3. How could you set up the experiment to find the experimental probability that at least 2 people in a group of 6 people will have the same birthday (same month and same date)? **Generate 6 random numbers from 1 to 365 for each trial.**

Teaching Tip **Math Background** Discuss with students that a greater number of trials leads to more accurate results.

Teaching Tip **Technology** The "random" numbers generated by a graphing calculator are determined by a "seed." Calculators of the same model return the same sequence of numbers for any given seed value, so it is possible for two or more students to generate the same "random" number sequence. The seed can be changed by storing a number for **rand.**

Technology LAB Organizer

Use with the lesson *Experimental Probability*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to find experimental probabilities by generating random numbers.

Materials: graphing calculator

INTERACTIVE **Online Edition**
Graphing Calculator, TechKeys

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

The number of possible outcomes determines the interval after **randInt(.** The number of people determines the number of times ENTER is pressed.

Close

Key Concept

Experiments can be modeled by using the random integer function on a calculator.

Assessment

Journal Have students explain how to use the random integer function to find the experimental probability that at least 3 people out of 5 were born in the same season.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.S.IC.1 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

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10-6 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Determine the theoretical probability of an event.

Convert between probabilities and odds.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

An experiment consists of spinning a spinner 8 times. The spinner lands on red 4 times, yellow 3 times, and green once. Find the experimental probability of each event.

1. The spinner lands on red. $\frac{1}{2}$
2. The spinner does not land on green. $\frac{7}{8}$
3. The spinner lands on yellow. $\frac{3}{8}$

Also available online

Math Humor

Q: What is the probability of a coin landing on heads 20 times out of 100 trials?

A: A lot higher than the probability that I'll stand around to find out.

COMMON CORE

CC.9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

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COMMON CORE

10-6

Theoretical Probability

CC.9-12.S.CP.1 Describe events as subsets of a sample space … using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ….

Objectives
Determine the theoretical probability of an event.

Convert between probabilities and odds.

Vocabulary
equally likely
theoretical probability
fair
complement
odds

Why learn this?
Theoretical probability can be used to determine the likelihood of different weather conditions. (See Example 2.)

A developing tornado near Amarillo, Texas.

When the outcomes in the sample space of an experiment have the same chance of occurring, the outcomes are said to be **equally likely**.

Equally likely outcomes

There is the same chance that the spinner will land on any of the colors.

Not equally likely outcomes

There is a greater chance that the spinner will land on blue than on any other color.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.

Theoretical Probability

$$\text{theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

An experiment in which all outcomes are equally likely is said to be **fair**. You can usually assume that experiments involving coins and number cubes are fair.

EXAMPLE 1 **Finding Theoretical Probability**

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

A **rolling a 3**

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{1}{6}$$

There is one 3 on a number cube.

$$= 0.1\overline{6}$$

$$= 16\frac{2}{3}\%$$

Caution!
The use of the word *experiment* does not necessarily mean you are looking for experimental probability.

Digital Vision/Getty Images

1 Introduce

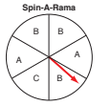
EXPLORATION

Theoretical Probability

At a school carnival, visitors have a chance to win a prize by spinning one of these spinners.

Spin to Win

Spin-A-Rama



1. Complete the tables. For each spinner, count the number of sectors labeled with each letter. Then find the fraction of the spinner labeled with each letter.

Spin to Win		
Sector Label	Number of Sectors	Fraction of Spinner
A	2	$\frac{2}{6} = \frac{1}{3}$
B	2	$\frac{1}{3}$
C	2	$\frac{1}{3}$

Spin-A-Rama		
Sector Label	Number of Sectors	Fraction of Spinner
A	2	$\frac{1}{3}$
B	3	$\frac{1}{2}$
C	1	$\frac{1}{6}$

2. For Spin to Win, is the spinner equally likely to land on A, B, and C? How about for Spin-A-Rama? yes; no

THINK AND DISCUSS

3. **Discuss** which result is most likely when you spin Spin-A-Rama. B is most likely because the largest fraction of the spinner is

Motivate

Have students pretend they are entering a contest in which the odds of winning are 3 to 1. Ask students to guess whether this means it will be impossible, unlikely, as likely as not, likely, or certain that they will win. Have students discuss what they think *odds* means. Tell students that in this section they will learn how to convert odds to probability to determine the likelihood of winning the contest.

Explorations and answers are provided in your online resources.

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

B **rolling a number greater than 3**

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{3}{6} = \frac{1}{2}$$ *There are 3 numbers greater than 3.*

$$= 0.5 = 50\%$$

CHECK IT OUT! **An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.**

1a. rolling an even number 50%

1b. rolling a multiple of 3 $33\frac{1}{3}\%$

Reading Math

The probability of an event can be written as *P*(event). *P*(heads) means "the probability that heads will be the outcome."

When you toss a coin, there are two possible outcomes, heads or tails. The table below shows the theoretical probabilities and experimental results of tossing a coin 10 times.

	P(heads)	*P*(tails)	*P*(heads) + *P*(tails)
Experimental Probability	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1$
Theoretical Probability	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

The sum of the probability of heads and the probability of tails is 1, or 100%. This is because it is certain that one of the two outcomes will always occur.

$$P(\text{event happening}) + P(\text{event not happening}) = 1$$

The **complement** of an event is all the outcomes in the sample space that are not included in the event. The sum of the probabilities of an event and its complement is 1, or 100%, because the event will either happen or not happen. Two events are *complementary events* if the sum of their probabilities equals 1 and if one event occurs if and only if the other does not.

$$P(\text{event}) + P(\text{complement of event}) = 1$$

EXAMPLE 2 Finding Probability by Using the Complement

The weather forecaster predicts a 20% chance of snow. What is the probability that it will not snow?

Afo Foto Agency/Alamy

$P(\text{snow}) + P(\text{not snow}) = 100\%$ *Either it will snow or it will not snow.*

$20\% + P(\text{not snow}) = 100\%$

$-20\% \qquad -20\%$ *Subtract 20% from both sides.*

$P(\text{not snow}) = 80\%$

CHECK IT OUT! **2.** A jar has green, blue, purple, and white marbles. The probability of choosing a green marble is 0.2, the probability of choosing blue is 0.3, and the probability of choosing purple is 0.1. What is the probability of choosing white? 0.4

COMMON ERROR ALERT

In **Example 1A,** some students may automatically write that the probability of rolling a 3 is $\frac{3}{6}$, instead of $\frac{1}{6}$. Remind them that the numerator is found by counting the number of ways to get a 3.

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Additional Examples

Example 1

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

A. rolling a 5 $16\frac{2}{3}\%$

B. rolling an odd number 50%

C. rolling a number less than 3 $33\frac{1}{3}\%$

Example 2

A box contains only red, black, and white blocks. The probability of choosing a red block is $\frac{1}{4}$, the probability of choosing a black block is $\frac{1}{2}$. What is the probability of choosing a white block? $\frac{1}{4}$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- When will the numerator and denominator of a probability be the same number?
- Why is it impossible for a probability to be an improper fraction?

EXAMPLE 2

- Why do you subtract from 1, or 100%, to find the complement?
- How are complements in probability similar to complements in geometry?

2 Teach

Guided Instruction

Emphasize that the formula for theoretical probability in this lesson applies to equally likely outcomes, while experimental probability depends on the frequency of a particular outcome and the number of trials. When converting between probabilities and odds, show that the sum of the two numbers in the odds ratio equals the total number of outcomes, which is the denominator in the probability fraction.

Reaching All Learners

Through Home Connection

Have students find advertisements for contests and sweepstakes. For each, have students use the odds (usually found in the small print at the end of the rules) to calculate the probability of winning.

Have students discuss their findings. Remind them that some prizes state "win up to...." For these, the odds of winning usually refer to the smallest amount possible.

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Additional Examples

Example 3

A. The probability of rolling a 2 on a number cube is $\frac{1}{6}$. What are the odds of rolling a 2? 1:5

B. The odds in favor of winning a contest are 1:9. What is the probability of winning the contest? $\frac{1}{10}$

C. The odds against a spinner landing on red are 2:3. What is the probability of the spinner landing on red? $\frac{3}{5}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How do you find the odds against an event if you know the odds in favor of the event?
- How do you find the probability of an event if you know the odds in favor of the event?

Odds are another way to indicate the likelihood of an event. **Odds** express likelihood by comparing the number of ways an event can happen to the number of ways an event can fail to happen. The *odds in favor of an event* describe the likelihood that the event will occur. The *odds against an event* describe the likelihood that the event will not occur.

Odds are usually written with a colon in the form $a:b$, but can also be written as a to b or $\frac{a}{b}$.

Know it! Note — Odds

ODDS IN FAVOR OF AN EVENT WITH EQUALLY LIKELY OUTCOMES

$$\text{odds in favor} = \frac{\text{number of ways an event can happen}}{\text{number of ways an event can fail to happen}} = a:b$$

ODDS AGAINST AN EVENT WITH EQUALLY LIKELY OUTCOMES

$$\text{odds against} = \frac{\text{number of ways an event can fail to happen}}{\text{number of ways an event can happen}} = b:a$$

a represents the number of ways an event can occur.

b represents the number of ways an event can fail to occur.

The two numbers given as the odds will add up to the total number of possible outcomes. You can use this relationship to convert between odds and probabilities.

EXAMPLE 3 Converting Between Odds and Probabilities

A **The probability of choosing a red card from a standard deck of playing cards is 50%. What are the odds of choosing a red card?**

The probability of choosing a red card is 50%, so the probability of not drawing a red card is 100% − 50% = 50%.

Odds in favor = 50:50, or 1:1

The odds in favor of choosing a red card are 1:1.

Reading Math

You may see an outcome called "favorable." This does not mean that the outcome is good or bad. A favorable outcome is the outcome you are looking for in a probability experiment.

B **The odds against choosing a green marble from a bag are 5:3. What is the probability of choosing a green marble?**

The odds against green are 5:3, so the odds in favor of green are 3:5. This means there are 3 favorable outcomes and 5 unfavorable outcomes for a total of 8 possible outcomes.

$$\frac{\text{number of ways event can happen}}{\text{total possible outcomes}} = \frac{3}{8}$$

The probability of choosing a green marble is $\frac{3}{8}$.

CHECK IT OUT!

3. The odds in favor of winning a free drink are 1:24. What is the probability of winning a free drink?

3. $\frac{1}{25}$

Victoria Smith/HMH

3 Close

Summarize

A bag of marbles holds 3 red, 4 blue, 5 yellow, and 6 white marbles. If one marble is drawn at random, find:

- P(blue) $\frac{2}{9}$
- P(not white) $\frac{2}{3}$
- odds in favor of yellow marble 5:13
- odds against red marble 5:1

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

MATHEMATICAL PRACTICES

THINK AND DISCUSS

Know it! Note

1. Tell how to find the probability of the complement of an event.
2. **GET ORGANIZED** Copy and complete the graphic organizer using the spinner.

Probabilities on Spinner	
P(gray)	
P(not gray)	
Odds in favor of gray	
Odds against gray	

Answers to *Think and Discuss*

1. Subtract the probability of the event from 1.
2. See Additional Answers.

10-6 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** All of the outcomes in the sample space that are not included in the event are called the ___?___. (*theoretical probability*, *complement*, or *odds*) complement

SEE EXAMPLE 1

Find the theoretical probability of each outcome.

2. rolling a number divisible by 3 on a number cube $33\frac{1}{3}\%$
3. tossing 2 coins and both landing with tails showing 25%
4. randomly choosing the letter S from the letters in STARS 40%
5. rolling a prime number on a number cube $\frac{1}{2}$

SEE EXAMPLE 2

6. A spinner is green, red, and blue. The probability that the spinner will land on green is 15% and red is 35%. What is the probability the spinner will land on blue? 50%

7. The probability of choosing a red marble from a bag is $\frac{1}{3}$. What is the probability of not choosing a red marble? $\frac{2}{3}$
8. You have a $\frac{1}{50}$ chance of winning. What is the probability you will not win? $\frac{49}{50}$
9. There is a $\frac{1}{10}$ chance that you will be chosen as class representative. What is the probability that you will not be chosen? $\frac{9}{10}$

SEE EXAMPLE 3

10. The odds against a spinner landing on blue are 3:1. What is the probability of the spinner landing on blue? $\frac{1}{4}$
11. The probability of choosing an ace from a deck of cards is $\frac{1}{13}$. What are the odds of choosing an ace? 1:12
12. The probability of not winning a game is 80%. What are the odds of winning? 1:4
13. The odds in favor of a spinner landing on blue are 1:3. What is the probability of landing on blue? $\frac{1}{4}$

10-6 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

Basic 14–28, 32–37
Average 14–22, 24–34, 35–38
Advanced 14–22, 23–29, 31–38

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 18, 22, 26, 34

Also available online

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 32, 34, 38

Construct viable arguments and critique the reasoning of others. Exercises 29–31, 33

Learn It Online
State Resources Online

Teaching Tip **Math Background** For **Exercise 29,** remind students that "less than 3" does not include 3. Likewise, the sample space for "greater than 3" is 4, 5, and 6.

MULTI-STEP TEST PREP **Exercise 32** involves finding theoretical probabilities. This exercise prepares students for the Multi-Step Test Prep.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–16	1
17–19	2
20–22	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Find the theoretical probability of each outcome.

14. rolling a 5 on a number cube $16\frac{2}{3}\%$
15. tossing 2 coins and 1 landing with heads showing, the other with tails showing 50%
16. randomly choosing a blue marble from a bag of 5 blue marbles, 8 red marbles, and 7 yellow marbles 25%
17. The probability of a spinner landing on yellow is $\frac{4}{9}$. What is the probability of it not landing on yellow? $\frac{5}{9}$

18. There is a 3% probability of winning a game. Find the probability of not winning the game. 97%
19. There is a 15% chance it will snow and a 15% chance it will rain. What is the probability that it will neither snow nor rain? 70%
20. The odds against winning a contest are 99:1. What is the probability of not winning the contest? $\frac{99}{100}$
21. The odds of choosing a white marble from a bag are 1:9. Find the probability of not choosing a white marble. $\frac{9}{10}$
22. The probability of a spinner landing on green is 25%. What are the odds of the spinner not landing on green? 3:1

Use the spinner for Exercises 23–28.

23. P(red) $\frac{1}{5}$
24. P(green) $\frac{1}{5}$
25. P(not blue) $\frac{4}{5}$
26. odds in favor of yellow 1:4
27. odds against red 4:1
28. odds against green 4:1

29. **Write About It** A number cube is rolled. Which event has a greater theoretical probability: rolling a number less than 3 or rolling a number greater than three? Explain.

29. Rolling a number greater than 3 because there are 3 outcomes in this event (4, 5, 6), whereas rolling a number less than 3 has 2 outcomes (1, 2).

30. ERROR ANALYSIS The odds in favor of an event are 1:4. Two students converted these odds into the probability of the event NOT happening. Which is incorrect? Explain the error.

A

Odds in favor of event	1:4
Probability that event will not happen	$\frac{1}{5}$

B

Odds in favor of event	1:4
Probability that event will not happen	$\frac{4}{5}$

30. Student A; since the odds in favor are 1:4, there is 1 way the event can happen and 4 ways the event can fail to happen. So the probability of the event not happening is $\frac{4}{5}$.

31. **Critical Thinking** The odds in favor of a certain event are the same as the odds against that event. What is the probability of the event occurring? $\frac{1}{2}$

MULTI-STEP TEST PREP

32. Chutes and Ladders is a children's game that uses a spinner with the numbers 1 through 6.
    a. What is the probability of spinning a 3? $\frac{1}{6}$
    b. What is the probability of spinning an odd number? $\frac{1}{2}$
    c. What is the probability of spinning a number that is less than or equal to 4? $\frac{2}{3}$

Sam Dudgeon/HMH

PRACTICE A
PRACTICE C
PRACTICE B

Name _____ Date _____ Class _____

LESSON 10-6 **Practice B**
Theoretical Probability

Find the theoretical probability of each outcome.

1. rolling a number less than 4 on a standard number cube 50%
2. randomly choosing a day of the week and it is a weekend $28\frac{4}{7}\%$
3. spinning red on a spinner with equal sections of red, blue, and green $33\frac{1}{3}\%$
4. randomly choosing the letter N from the letters in NUMBER $16\frac{2}{3}\%$
5. The probability it will snow is 60%. What is the probability it will not snow? 40%
6. The probability of tossing two coins and having them land heads up is $\frac{1}{4}$. What is the probability the coins will not land heads up? $\frac{3}{4}$
7. A spinner has red, green, blue, and yellow. The probability of spinning a red is 0.4, the probability of spinning a blue is 0.05 and the probability of spinning a yellow is 0.25. What is the probability of spinning a green? 0.3
8. Miguel entered a contest offering prizes to the top 3 finishers. The probability of winning 1st is 12%, the probability of winning 2nd is 18% and probability of winning 3rd is 20%. What is the probability that Miguel will not win any prize? 50%
9. The odds of winning a contest are 1:50. What is the probability of winning the contest? $\frac{1}{51}$
10. The odds against a spinner landing on yellow are 3:1. What is the probability the spinner will not land on yellow? $\frac{3}{4}$
11. The probability of a thunderstorm is 80%. What are the odds that there will be a thunderstorm? 4:1
12. The odds of selecting a red card from a box of cards are 2:5. What is the probability of not selecting a red card from a box? $\frac{5}{7}$

The table shows how many of each letter are in a bag. Use the table for 13–16. Find the following.

Letter	How Many in Bag
A	5

13. P(A)
14. P(B)

READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 10-6 **Reading Strategies**
Using a Model

Use the model below as a guide to finding probabilities and odds.

Experiment: Choose a letter at random from the word MODEL.

Probability of choosing a vowel: $P(\text{vowel}) = \frac{\#\text{ of vowels}}{\text{total }\#\text{ of letters}}$	Probability of choosing a vowel: $P(\text{not vowel}) = \frac{\#\text{ of non-vowels}}{\text{total }\#\text{ of letters}}$
$\frac{\{O, E\}}{\{M, O, D, E, L\}}$	$\frac{\{M, D, L\}}{\{M, O, D, E, L\}}$
$\frac{2}{5}$	$\frac{3}{5}$

MODEL

Odds in favor of choosing a vowel:	Odds against choosing a vowel:
# of vowels : # of non-vowels	# of non-vowels : # of vowels
{O, E} : {M, D, L}	{M, D, L} : {O, E}
2:3	3:2

Use the information above to complete the following.

1. Look at the probability of choosing a vowel and the probability of not choosing a vowel.
   a. What is true about the denominators? they are the same
   b. How are the numerators related to the denominator?
   The sum of the numerators equals the denominator: 2 + 3 = 5.
2. Compare the probability of choosing a vowel and the odds in favor of choosing a vowel.
   The numerator in the probability is the same as the first number in the odds ratio. The numbers in the odds ratio sum to the denominator in the probability.

There is a prize in 1 cereal box out of every 5 cereal boxes. You buy 1 box. Find each of the following.

1 1

RETEACH

Name _____ Date _____ Class _____

LESSON 10-6 **Review for Mastery**
Theoretical Probability

When the outcomes in the sample space of an experiment have the same chance of occurring, the outcomes are **equally likely**.

$$\text{theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

A bag of marbles contains 8 yellow, 2 red, and 10 green marbles. An experiment consists of selecting one marble at random from the bag. Find the theoretical probability of each outcome.

A. **selecting a yellow marble**

Step 1: Determine total number of outcomes.
$8 + 2 + 10 = 20$
Step 2: Determine the number of ways the event can occur.
The event occurs if a yellow marble is selected. Thus, the number of ways the event can occur is 8.
Step 3: Find theoretical probability.
$P(\text{yellow}) = \frac{8}{20} = 0.4 = 40\%$

B. **selecting a red or yellow marble**

Step 1: Determine total number of outcomes.
$8 + 2 + 10 = 20$
Step 2: Determine the number of ways the event can occur.
The event occurs if a red or yellow marble is chosen. Thus the number of ways the event can occur is $2 + 8 = 10$.
Step 3: Find theoretical probability.
$P(\text{red or yellow}) = \frac{10}{20} = 0.5 = 50\%$

The sum of the probabilities of an event and its complement is 100%, because the event will either happen or not happen.

Use the information above to find the probability of NOT selecting a yellow marble.

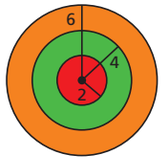
$P(\text{yellow}) + P(\text{not yellow}) = 100\%$
$40\% + P(\text{not yellow}) = 100\%$
$-40\% \quad\quad -40\%$
$P(\text{not yellow}) = 60\%$

Find the theoretical probability of each outcome.

1. rolling an odd number on a number cube $\frac{5}{7}$
2. randomly choosing a red marble from a bag of 6 red, 6 blue and 8 green marbles $\frac{5}{7}$
3. rolling a number less than or equal to 2 on a number cube $\frac{5}{7}$ $\frac{5}{7}$

33. Write About It Explain how to convert odds to probability.

34. Geometry The radius of each circle in the diagram is given. Find the probability that a point chosen at random will lie in the red area of the diagram. (*Hint:* Find the ratio of the red area to the total area of the diagram.) $\frac{1}{9}$



TEST PREP

35. Two coins are tossed. What is the probability that at least one of the coins lands with heads showing?

Ⓐ 25% Ⓑ $33\frac{1}{3}\%$ Ⓒ 50% Ⓓ 75%

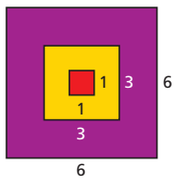
36. A standard number cube is rolled. Which has the greatest probability?

Ⓕ *P*(even) Ⓖ *P*(less than 5) Ⓗ *P*(not 2) Ⓙ *P*(greater than 3)

37. Find the probability that a point chosen at random would fall in the yellow area.

Ⓐ $\frac{1}{6}$ Ⓒ $\frac{4}{9}$

Ⓑ $\frac{2}{9}$ Ⓓ $\frac{2}{3}$



CHALLENGE AND EXTEND

Use the results of 3 coin-tossing experiments in the table for Exercise 38.

38. Find the experimental probability of the coin showing heads for

a. experiment 1. 0.4

b. experiment 2. 0.48

c. experiment 3. 0.502

d. Find the theoretical probability of the coin showing heads. 0.5

	Experiment		
	1	2	3
Number of Tosses	10	100	1000
Heads	4	48	502

e. Write About It How do the experimental probabilities of each experiment compare to the theoretical probability? The exp. prob. is closer to the theor. prob. in the experiments with more tosses.

TEST PREP DOCTOR For **Exercise 35,** encourage students to write the sample space as (HH, HT, TH, TT). Also remind students that *at least one head* means either one or two heads.

Answers

33. If the odds in favor of an event are $a:b$, then there are a total of $a + b$ outcomes, where a is the number of ways the event can occur. So the probability of the event is $\frac{a}{a+b}$.

Journal

Have students explain the difference between theoretical and experimental probability.

ALTERNATIVE ASSESSMENT

Have students write a probability problem using a number cube (MK) or a fair spinner (MK) with four different colors. Have students choose an outcome, give the probability of the outcome, and convert the probability to odds in favor of the event.

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10-6 Lesson Quiz

Find the theoretical probability of each outcome.

1. randomly choosing *B* from the letters in **ALGEBRA** $\frac{1}{7}$

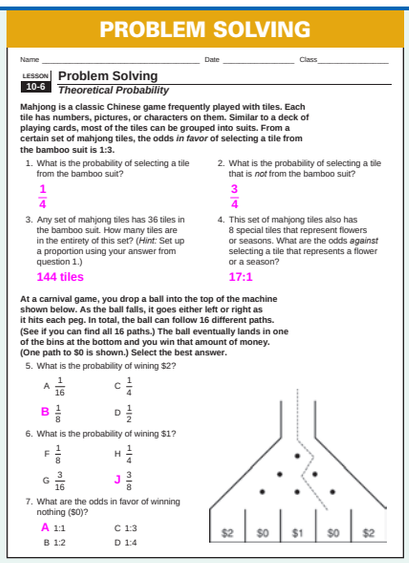
2. rolling a factor of 10 on a number cube $\frac{1}{2}$

3. The probability that it will be sunny is 15%. What is the probability that it will not be sunny? 85%

4. The probability of choosing a red marble out of a bag of marbles is $\frac{3}{10}$. What are the odds in favor of choosing a red marble? 3:7

5. A spinner has only blue and red sections. The odds against the spinner landing on blue are 7:5. What is the probability of the spinner landing on red? $\frac{5}{12}$

Also available online


PROBLEM SOLVING

Name _____ Date _____ Class _____

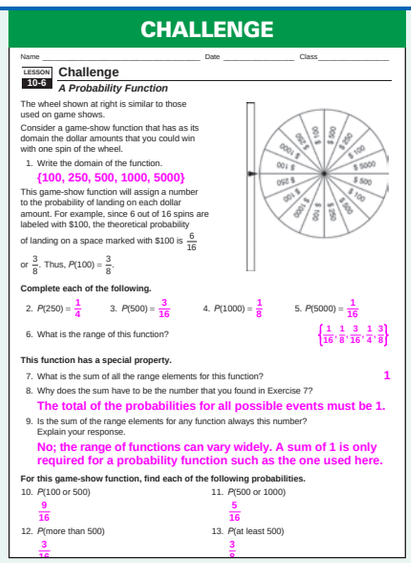
LESSON 10-6 **Problem Solving**
Theoretical Probability

Mahjong is a classic Chinese game frequently played with tiles. Each tile has numbers, pictures, or characters on them. Similar to a deck of playing cards, most of the tiles can be grouped into suits. From a certain set of mahjong tiles, the odds *in favor* of selecting a tile from the bamboo suit is 1:3.

1. What is the probability of selecting a tile from the bamboo suit? $\frac{1}{4}$
2. What is the probability of selecting a tile that is *not* from the bamboo suit? $\frac{3}{4}$
3. Any set of mahjong tiles has 36 tiles in the bamboo suit. How many tiles are in the entirety of this set? (*Hint:* Set up a proportion using your answer from question 1.) 144 tiles
4. This set of mahjong tiles also has 8 special tiles that represent flowers or seasons. What are the odds *against* selecting a tile that represents a flower or a season? 17:1

At a carnival game, you drop a ball into the top of the machine shown below. As the ball falls, it goes either left or right as it hits each peg. In total, the ball can follow 16 different paths. (See if you can find all 16 paths.) The ball eventually lands in one of the bins at the bottom and you win that amount of money. (One path to $0 is shown.) Select the best answer.

5. What is the probability of wining $2?
   A $\frac{1}{16}$ C $\frac{1}{4}$
   B $\frac{1}{8}$ D $\frac{1}{2}$
6. What is the probability of wining $1?
   F $\frac{1}{8}$ H $\frac{1}{4}$
   G $\frac{3}{16}$ J $\frac{3}{8}$
7. What are the odds in favor of winning nothing ($0)?
   A 1:1 C 1:3
   B 1:2 D 1:4


CHALLENGE

Name _____ Date _____ Class _____

LESSON 10-6 **Challenge**
A Probability Function

The wheel shown at right is similar to those used on game shows.

Consider a game-show function that has as its domain the dollar amounts that you could win with one spin of the wheel.

1. Write the domain of the function. {100, 250, 500, 1000, 5000}

This game-show function will assign a number to the probability of landing on each dollar amount. For example, since 6 out of 16 spins are labeled with $100, the theoretical probability of landing on a space marked with $100 is $\frac{6}{16}$ or $\frac{3}{8}$. Thus, $P(100) = \frac{3}{8}$.

Complete each of the following.

2. $P(250) = \frac{1}{4}$ 3. $P(500) = \frac{3}{16}$ 4. $P(1000) = \frac{1}{8}$ 5. $P(5000) = \frac{1}{16}$
6. What is the range of this function? $\left\{\frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}\right\}$

This function has a special property.

7. What is the sum of all the range elements for this function? 1
8. Why does the sum have to be the number that you found in Exercise 7? The total of the probabilities for all possible events must be 1.
9. Is the sum of the range elements for any function always this number? Explain your response. No; the range of functions can vary widely. A sum of 1 is only required for a probability function such as the one used here.

For this game-show function, find each of the following probabilities.

10. P(100 or 500) $\frac{9}{16}$
11. P(500 or 1000) $\frac{5}{16}$
12. P(more than 500) $\frac{3}{16}$
13. P(at least 500) $\frac{3}{8}$

10-7 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Find the probability of independent events.

Find the probability of dependent events.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find the theoretical probability of each outcome.

1. rolling a 6 on a number cube $\frac{1}{6}$
2. rolling an odd number on a number cube $\frac{1}{2}$
3. tossing a coin and it landing heads up $\frac{1}{2}$

Also available online

Math Humor

Q: What do you call two unrelated activities scheduled on July 4th?

A: Independence events.

COMMON CORE

CC.9-12.S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

CC.9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

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COMMON CORE

10-7 Independent and Dependent Events

CC.9-12.S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities … *Also* **CC.9-12.S.CP.6, CC.9-12.S.CP.3, CC.9-12.S.CP.5, CC.9-12.S.CP.1**

Objectives
Find the probability of independent events.

Find the probability of dependent events.

Vocabulary
independent events
dependent events

Why learn this?
You may need to understand independent and dependent events to determine the number of reading selections available.

Adam's teacher gives the class two lists of titles and asks each student to choose two of them to read. Adam can choose one title from each list or two titles from the same list.

One title from each list

Animal Farm	Never Cry Wolf
Ethan Frome	Night
Frankenstein	Things Fall Apart
Great Expectations	Wish You Well
Jane Eyre	Wuthering Heights

Choosing a title from one list does not affect the number of titles to choose from on the other list. The events are *independent.*

Two titles from the same list

Animal Farm	~~Animal Farm~~
Ethan Frome	Ethan Frome
Frankenstein	Frankenstein
Great Expectations	Great Expectations
Jane Eyre	Jane Eyre

Choosing a title from one of the lists changes the number of titles that can be chosen from the same list. The events are *dependent.*

Events are **independent events** if the occurrence of one event does not affect the probability of the other. Events are **dependent events** if the occurrence of one event does affect the probability of the other.

EXAMPLE 1 **Classifying Events as Independent or Dependent**

Tell whether each set of events is independent or dependent. Explain your answer.

A **A dime lands heads up and a nickel lands heads up.**
The result of tossing a dime does not affect the result of tossing a nickel, so the events are independent.

B **You choose a colored game piece in a board game, and then your sister picks another color.**
Your sister cannot pick the same color you picked, and there are fewer game pieces for your sister to choose from after you choose, so the events are dependent.

1a. Indep.; the result of rolling the number cube the 1st time does not affect the result of the 2nd roll.

1b. Dep.; choosing the 1st student leaves fewer students to choose from the 2nd time.

CHECK IT OUT! **Tell whether each set of events is independent or dependent. Explain your answer.**

1a. A number cube lands showing an odd number. It is rolled a second time and lands showing 6.

1b. One student in your class is chosen for a project. Then another student in the class is chosen.

1 Introduce

EXPLORATION

Independent and Dependent Events

An experiment consists of spinning the spinner shown here and rolling a standard number cube.

1. Complete the table to show all of the possible outcomes. (In the table, B-1 means that the spinner landed on blue and that 1 was rolled on the number cube.)

	1	2	3	4	5	6
Blue	B-1	B-2	B-3	B-4	B-5	B-6
Green	G-1	G-2	G-3	G-4	G-5	G-6
Red	R-1	R-2	R-3	R-4	R-5	R-6

2. How many different outcomes are there? 18
3. How many of the outcomes involve the spinner landing on green and an even number being rolled on the number cube? 3
4. What is the probability of the spinner landing on green and an even number being rolled on the number cube? $\frac{3}{18} = \frac{1}{6}$
5. What is the probability of the spinner landing on green? What is the probability of rolling an even number on the number cube? $\frac{1}{3}, \frac{1}{2}$

THINK AND DISCUSS

6. **Explain** how the probability you found in Step 4 is related to the two probabilities you found in Step 5. The probability in Step 4 is the product of the probabilities in Step 5.

Motivate

Ask students for ideas about how to find the probability that three coins land showing heads. Have students find the sample space. HHH, HHT, HTH, THH, TTH, THT, HTT, TTT Then have them find the probability of all three coins showing heads. $\frac{1}{8}$

Show students that writing the entire sample space for some events can take time. Explain that in this section they will learn to find the probability of events without writing out the sample space.

Explorations and answers are provided in your online resources.

Student to Student

Independent and Dependent Events

Lydia Delgado
Northridge High School

I use the everyday meanings of independent and dependent to remember their mathematical meanings.

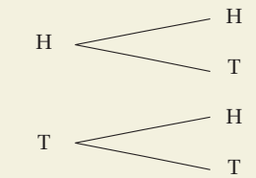
The math class I take next year depends on which math classes I have already taken.

For dependent events, the occurrence of one affects the probability of the other.

The history class I take next year is independent from (does not depend on) which math classes I have taken.

For independent events, the occurrence of one does not affect the occurrence of the other.

Suppose an experiment involves tossing two fair coins. The sample space of outcomes is shown by the tree diagram. Determine the theoretical probability of both coins landing heads up.



There are four possible outcomes in the sample space: $\{(H, H), (H, T), (T, H), (T, T)\}$

Only one outcome includes both coins landing heads up.

The theoretical probability of both coins landing heads up is $\frac{1}{4}$.

Now look back at the separate theoretical probabilities of each coin landing heads up. The theoretical probability in each case is $\frac{1}{2}$. The product of these two probabilities is $\frac{1}{4}$, the same probability shown by the tree diagram.

To determine the probability of two independent events, multiply the probabilities of the two events.

Know it! Note

Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 2 Finding the Probability of Independent Events

A **An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 7 blue marbles and 3 yellow marbles. What is the probability of selecting a yellow marble and then a blue marble?**

Because the first marble is replaced after it is selected, the sample space for each selection is the same. The events are independent.

$$P(\text{yellow, blue}) = P(\text{yellow}) \cdot P(\text{blue})$$
$$= \frac{3}{10} \cdot \frac{7}{10}$$
$$= \frac{21}{100}$$

The probability of selecting yellow is $\frac{3}{10}$, and the probability of selecting blue is $\frac{7}{10}$.

David Young-Wolff/Alamy

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Additional Examples

Example 1

Tell whether each set of events is independent or dependent. Explain your answer.

A. You select a card from a standard deck of cards and hold it. A friend selects another card from the same deck. Dependent; your friend cannot pick the card you picked and has fewer cards to choose from.

B. You toss a coin and it lands heads up. You toss the same coin and it lands heads up again. Independent; the result of the first toss does not affect the sample space for the second toss.

Example 2

A. An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 3 red marbles and 12 green marbles. What is the probability of selecting a red marble and then a green marble? $\frac{4}{25}$

B. A coin is tossed 4 times. What is the probability of getting 4 heads in a row? $\frac{1}{16}$

Also available online

2 Teach

Guided Instruction

Remind students that they have previously found the probability of one event. For two or more events, they have to know whether the events are independent or dependent. Emphasize that independent events have the same probability each time, regardless of past events. Thus, a coin that lands heads up 10 times in a row still has a 50% probability of landing heads up on the next toss. Coins can't "remember" what they did on previous tosses.

Reaching All Learners

Through Concrete Manipulatives

To help show the difference between dependent and independent events give each student (or pair of students) two red and two black marbles (or checker pieces or playing cards). Instruct them to choose two objects at random without replacing the first object, and record the results of ten trials. Then have them do the same with replacement. Find the experimental probability of selecting two red marbles with and without replacement. Discuss which probability is higher and why.

Mathematical Practices Questioning Strategies

EXAMPLE 1

- How are independent events different from dependent events?

EXAMPLE 2

- Does the probability of independent events change if the order of the occurrence of the events changes? Explain.

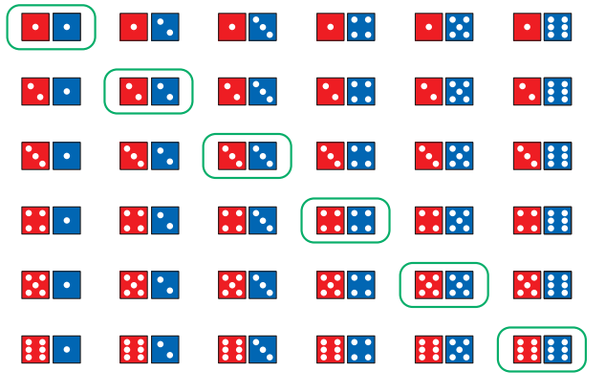
Teaching Tip **Inclusion** Explain that choosing two items with replacement represents independent events. Choosing without replacement represents dependent events.

Teaching Tip **Critical Thinking** Use **Example 2B** to show students that probabilities can be calculated for more than two events. For three independent events, the equation becomes $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$.

B **When a person rolls 2 number cubes and they land showing the same number, we say the person rolled doubles. What is the probability of rolling doubles 3 times in a row?**

The result of one roll does not affect any following rolls. The events are independent.

When you roll 2 number cubes, there are 36 possible outcomes, six of which are doubles:

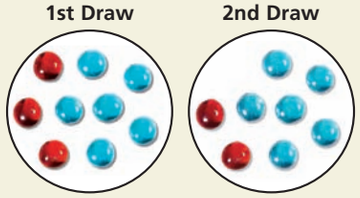


So, the probability of rolling doubles once is $P(\text{double}) = \frac{6}{36} = \frac{1}{6}$.

$$P(\text{double, double, double}) = P(\text{double}) \cdot P(\text{double}) \cdot P(\text{double})$$
$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$$
$$= \frac{1}{216}$$

CHECK IT OUT! **2.** An experiment consists of spinning the spinner twice. What is the probability of spinning two odd numbers? $\frac{1}{4}$

Suppose an experiment involves drawing marbles from a bag. Determine the theoretical probability of drawing a red marble and then drawing a second red marble *without replacing the first one.*



The sample space for the second draw is not the same as the sample space for the first draw. There are fewer marbles in the bag for the second draw.

This means the events are dependent.

Probability of drawing a red marble on the first draw $= \frac{3}{9} = \frac{1}{3}$

Probability of drawing a red marble on the second draw $= \frac{2}{8} = \frac{1}{4}$

(tl), Peter Van Steen/HMH; (bc), Victoria Smith/HMH

To determine the probability of two dependent events, multiply the probability of the first event times the probability of the second event after the first event has occurred.

Know it! Note

Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$.

EXAMPLE 3 Problem-Solving Application

There are 7 pink flowers and 5 yellow flowers in a bunch. Jane selects a flower at random, and then Leah selects a flower at random from the remaining flowers. What is the probability that Jane selects a pink flower and Leah selects a yellow flower?

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

1 Understand the Problem

The **answer** will be the probability that a yellow flower is chosen after a pink flower is chosen.

List the **important information:**

- Jane chooses a pink flower from 7 pink flowers and 5 yellow flowers.
- Leah chooses a yellow flower from 6 pink flowers and 5 yellow flowers.

2 Make a Plan

Draw a diagram.

Flowers Jane can choose from:

7 pink
5 yellow
12 total

Flowers Leah can choose from:

6 pink
5 yellow
11 total

After Jane selects a flower, the sample space changes. So the events are dependent.

3 Solve

$$P(\text{pink and yellow}) = P(\text{pink}) \cdot P(\text{yellow after pink})$$

$$= \frac{7}{12} \cdot \frac{5}{11}$$

Jane selects one of 7 pink flowers from 12 total flowers. Then Leah selects one of 5 yellow flowers from the 11 flowers left.

$$= \frac{35}{132}$$

The probability that Jane selects a pink flower and Leah selects a yellow flower is $\frac{35}{132}$.

4 Look Back

Drawing a diagram helps you see how the sample space changes. This means the events are dependent, so you can use the formula for probability of dependent events.

CHECK IT OUT!

3. A bag has 10 red marbles, 12 white marbles, and 8 blue marbles. Two marbles are randomly drawn from the bag. What is the probability of drawing a blue marble and then a red marble? $\frac{8}{87}$

Victoria Smith/HMH

COMMON ERROR ALERT

When finding the probability of dependent events in **Example 3,** students often forget that the denominator of the second fraction will be different from the first. Remind them that after the first event has occurred, there are fewer possible outcomes.

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Additional Examples

Example 3

A snack cart has 6 bags of pretzels and 10 bags of chips. Grant selects a bag at random, and then Iris selects a bag at random. What is the probability that Grant will select a bag of pretzels and Iris will select a bag of chips? $\frac{1}{4}$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- Why are the denominators different when calculating probability for dependent events?
- What is the meaning of $P(B \text{ after } A)$?

3 Close

Summarize

Have students find the probability for the following. A bag has 2 green, 4 red, and 3 white marbles.

1) Probability of choosing one green, replacing it, and choosing one red $\frac{8}{81}$
2) Probability of choosing one green and then one red, without replacement $\frac{1}{9}$

Discuss why the second situation has the higher probability.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose* Before the Lesson**
Warm Up, TE

***Monitor* During the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess* After the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: Choosing an ace from a deck of cards and then choosing a king; the events are dependent because the sample space changes after the first card is selected.
2. See Additional Answers.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Give an example of two events that are dependent. Explain why the events are dependent.
2. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

	Example	Probability
Dependent Events	$A =$ $B =$	$P(A \text{ and } B) =$
Independent Events	$A =$ $B =$	$P(A \text{ and } B) =$

10-7 Exercises

10-7 Exercises

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Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
Basic 14–22, 29, 35–38
Average 14–21, 23–39
Advanced 14–20 even, 23–44

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 18, 20, 26, 32

Answers

2. Dep.; the choice of the 1st card affects the sample space for the choice of the 2nd card.
3. Indep.; the guess for the 1st question does not affect the sample space for the 2nd question.
4. Dep.; after you have selected your kitten, there are fewer kittens for your friend to choose from.

5–6. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 2–23, 28–32, 39–44

Reason abstractly and quantitatively. Exercises 24–27

Construct viable arguments and critique the reasoning of others. Exercises 30, 33–34

Model with mathematics. Exercises 41–44

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GUIDED PRACTICE

1. **Vocabulary** Two events are _____?_____ if the occurrence of one event affects the probability of the other event. (*independent* or *dependent*) **dependent**

SEE EXAMPLE 1

Tell whether each set of events is independent or dependent. Explain your answers.

2. You draw a heart from a deck of cards and set it aside. Then you draw a club from the deck of cards.
3. You guess "true" on two true-false questions.
4. You select a kitten to adopt from an animal shelter. Then your friend selects a kitten to adopt.
5. You order from a menu, and then your friend orders.
6. A doctors' office schedules several patients. Then you make an appointment.

SEE EXAMPLE 2

7. A coin is tossed three times. What is the probability of the coin landing heads up three times? $\frac{1}{8}$
8. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two odd numbers? $\frac{16}{49}$
9. Stacey rolls two number cubes. What is the probability that the sum of the numbers on the two number cubes is 7? $\frac{1}{6}$
10. A number cube is rolled twice and a coin is tossed once. What is the probability of the coin landing heads up and the number cube landing with 2 showing both times? $\frac{1}{72}$
11. A spinner with four equal sections of red, yellow, green, and blue is spun twice. What is the probability that it lands on yellow and then on green? $\frac{1}{16}$

Sam Dudgeon/HMH

SEE EXAMPLE 3

12. A bag contains 4 red marbles, 3 white marbles, and 6 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting a white marble from the bag? $\frac{1}{13}$

13. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two odd numbers? $\frac{2}{7}$

14. There are 15 boys and 14 girls in a room. Two of them are selected at random to take a survey. What is the probability that the two people selected will be girls? $\frac{13}{58}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–16	1
17–19	2
20–22	3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Tell whether each set of events is independent or dependent. Explain your answer.

15. The teacher randomly selects two students from the class.

16. You roll a 3 on a number cube and choose a 3 from a deck of cards.

17. A number cube is rolled three times. What is the probability of rolling three even numbers? $\frac{1}{8}$

18. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two even numbers? $\frac{1}{4}$

19. Stacey rolls a number cube and tosses a coin. What is the probability that she rolls a 5 and the coin lands heads up? $\frac{1}{12}$

20. A bag contains 5 red marbles, 3 white marbles, and 4 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting another red marble from the bag? $\frac{5}{33}$

21. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two even numbers? $\frac{2}{9}$

22. A game has 6 colored playing pieces. They are red, yellow, green, blue, purple, and white. You and a friend each pick a game piece without looking. What is the probability that your friend picks the blue piece and then you pick the yellow piece? $\frac{1}{30}$

LINK **School**

On some standardized tests, there is no penalty for a wrong answer. On these tests, it is better to guess than to leave the answer blank, especially if some choices can be eliminated.

23. **School** On a multiple-choice test, each question has 4 possible answers. A student does not know the answers to three questions, so the student guesses.

a. What is the probability that the student gets all three questions wrong? $\frac{27}{64}$

b. What is the probability that the student gets all three questions right? $\frac{1}{64}$

Tell whether each set of events is independent or dependent. Explain your answer.

24. Pick "Joe" from a box of names, replace it, and then pick "Craig."

25. Pick "Joe" from a box of names, set it aside, and then pick "Craig."

26. Roll a prime number on a number cube and get tails when tossing a coin.

27. Roll an even number, then an odd number, and then a 1 on a number cube.

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COMMON ERROR ALERT

In **Exercise 23,** students might write $\frac{1}{64}$ for both **a** and **b.** Remind students that the numerator in a probability fraction represents the number of ways that the event can occur. With four possible answers, there are three ways to choose the wrong answer and one way to choose the right answer.

Answers

15. Dependent; the choice of the first student affects the sample space for the choice of the second student.

16. Independent; the result of rolling the number cube does not affect the sample space of the deck of cards.

24. Independent; the choice of the first name does not affect the sample space for the choice of the second name.

25. Dependent; the choice of the first name affects the sample space for the choice of the second name.

26. Independent; the roll of the number cube does not affect the sample space for tossing the coin.

27. Independent; each roll does not affect the sample space for the next roll.

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State Resources Online

MULTI-STEP TEST PREP **Exercise 28** involves finding the probability of independent events. This exercise prepares students for the Multi-Step Test Prep.

MULTI-STEP TEST PREP

28. Yahtzee is a game that involves rolling five dice. On his or her turn, a player can roll up to three times to try to score points in various categories. Rolling a "Yahtzee" means rolling five of a kind, or five of the same number.

a. Juan has rolled twice and has three 5's showing. He rolls the remaining two dice. What is the probability that both dice will land showing 5? $\frac{1}{36}$

b. Shauna has two 3's showing. She has one more roll with the remaining three dice. What is the probability that all three dice will land showing 3? $\frac{1}{216}$

c. Mike rolls all five number cubes and all of them land showing 6. What is the probability of getting five 6's in one roll? $\frac{1}{7776}$

29. A bag contains 3 red, 5 blue, and 2 white marbles.

a. Find the probability of randomly picking a red marble, replacing it, and then picking a blue marble. $\frac{3}{20}$

b. Find the probability of randomly picking a red marble, setting it aside, and then picking a blue marble. $\frac{1}{6}$

c. Find the probability of randomly picking a red marble, replacing it, and then picking another red marble. $\frac{9}{100}$

d. Find the probability of randomly picking a red marble, setting it aside, and then picking another red marble. $\frac{1}{15}$

30. **Entertainment** Joe and Maria are playing a board game. On each turn, the player rolls two number cubes. Both players have two turns remaining.

a. Joe will win if he rolls double 6's on both turns. What is the probability that Joe will roll double 6's on both turns? $\frac{1}{1296}$

b. Maria will win if she rolls 2 on the first turn and 12 on the second turn. What is the probability that Maria will roll 2 on the first turn and 12 on the second turn?

30b. $\frac{1}{1296}$

c. **Write About It** Who has the better probability of winning? Explain.

c. The players have the same prob. of winning.

31. Tamika has $2.50 in quarters in her pocket, including four state quarters. She reaches into her pocket and takes out two quarters, one at a time. What is the probability that they are both state quarters? $\frac{2}{15}$

32. Ten cards are numbered 1 through 10 and placed in a bag. You draw a card, set it aside, and draw another card. What is the probability that you will draw two numbers that are divisible by 3? $\frac{1}{15}$

33. **Critical Thinking** What is the probability of a coin landing heads up on two tosses if it lands tails up on the first toss? Explain.

33. The prob. is 0 because it is impossible for the coin to land heads up on both tosses.

34. **Write About It** Explain what it means for two events to be independent.

2 events are indep. if the occurrence of 1 event does not affect the prob. of the other.

TEST PREP

35. A number cube is rolled twice. What is the probability of getting a 2 on both rolls?

Ⓐ $\frac{1}{3}$ Ⓑ $\frac{1}{4}$ Ⓒ $\frac{1}{9}$ Ⓓ $\frac{1}{36}$

36. In baseball, Julio averages 3 hits in every 10 at bats. What is the probability that he will get hits in both of his next two at bats?

Ⓕ 0.03 Ⓖ 0.09 Ⓗ 0.3 Ⓙ 0.9

Sam Dudgeon/HMH

PRACTICE A

PRACTICE C

PRACTICE B

Name _____ Date _____ Class _____

LESSON 10-7 **Practice B**

Independent and Dependent Events

Tell whether each set of events is independent or dependent. Explain your answer.

1. You roll a die and flip a coin. independent
the result of the die does not affect the result of the coin.
2. You select one marble, do not replace it, then select another marble. dependent
selecting one marble without replacing it will change the number to choose from.
3. A number cube is rolled three times. What is the probability of rolling a 2 each time? $\frac{1}{216} \approx 0.5\%$
4. The numbers 1 – 40 are written on pieces of paper and put in a box. Two pieces of paper are randomly selected. What is the probability both numbers will be multiples of 4? $\frac{3}{52} \approx 5.8\%$
5. A coin is tossed 4 times. What is the probability of getting 4 tails? $\frac{1}{16} = 6.25\%$
6. **A bag contains 2 yellow, 12 red, and 6 green marbles.**
   a. What is the probability of selecting a red marble, replacing it, then selecting another red marble? $\frac{9}{25} = 36\%$
   b. What is the probability of selecting a red marble, not replacing it, then selecting another red marble? $\frac{33}{95} \approx 34.7\%$
   c. What is the probability of selecting 1 yellow marble, not replacing it, then selecting a green marble? $\frac{3}{95} \approx 3.2\%$
7. **There are 7 girls and 3 boys in a class. Two students are to be randomly chosen for a special project.**
   a. What is the probability both students will be girls? $\frac{7}{15} \approx 46.7\%$
   b. What is the probability both students will be boys? $\frac{1}{15} \approx 6.7\%$
   c. What is the probability of selecting a boy and a girl? $\frac{7}{30} \approx 23.3\%$

A music class consists of 9th and 10th graders as shown in the table. Two students will be selected at the same time.

Music Class		
	9th	10th
male	9	8
female	12	11

8. What is the probability both students are male? $\frac{34}{190} \approx 17.4\%$

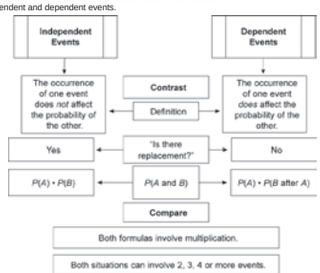
READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 10-7 **Reading Strategies**

Compare and Contrast

The chart below shows the similarities and differences between independent and dependent events.



Identify the following as independent or dependent events.

1. Ben rolls a 3 on a number cube. Then he rolls a 4. independent
2. May chooses a prize out of a bag and keeps it. Then Grace chooses a prize. dependent
3. Greg picks a red crayon out of a box, uses it, and then puts it back. Then Jo picks a red crayon. independent

A bag contains 5 blue marbles, 3 red marbles, and 2 green marbles. Find the probability of each event.

4. P(blue, blue) with replacement
5. P(blue, blue) without replacement
6. P(blue, red, green) with replacement

RETEACH

Name _____ Date _____ Class _____

LESSON 10-7 **Review for Mastery**

Independent and Dependent Events

Events are **independent** if the occurrence of one event does not affect the probability of the other.

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

In other words, if two or more events are independent, multiply their individual probabilities.

An experiment consists of rolling a number cube twice. What is the probability of rolling a 3 the first time and a 2 the second time?

Step 1: Determine if the two events are independent.

The first roll of a number cube will not affect the second roll. Therefore, the two rolls are independent events.

Step 2: Find the probability.

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(3, 2) = P(3) \cdot P(2)$

$= \frac{1}{6} \cdot \frac{1}{6}$

$= \frac{1}{36}$

An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 2 red marbles, 1 white marble, and 7 yellow marbles. What is the probability of selecting a white marble and then a yellow marble?

Step 1: Determine if the two events are independent.

Because the first marble is replaced, the probability of getting a certain color on the second selection is not affected by which color was selected first. The events are independent.

Step 2: Find the probability.

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(\text{white, yellow}) = P(\text{white}) \cdot P(\text{yellow})$

$= \frac{1}{10} \cdot \frac{7}{10} = \frac{7}{100}$

1. Six cards are numbered 1 to 6 and placed in a box. One card is selected at random and replaced. Another card is selected at random. What is the probability of selecting a 1 and then a 6?
   $P(1) \frac{1}{6}$, $P(6) \frac{1}{6}$, $P(1 \text{ and } 6) \frac{1}{36}$
2. A coin is tossed three times. What is the probability of the coin landing heads all three times? $\frac{1}{8}$
3. A bag contains 5 red marbles, 8 white marbles, and 7 green marbles. What is the probability of randomly selecting a white marble, replacing it, then randomly selecting another white marble? $\frac{4}{25}$
4. Ten cards are numbered from 1 to 10 and placed in a box.

37. Two people from a group of 30 will be selected at random for a prize. Twenty people in the group are women. What is the probability that both people selected will be men?

Ⓐ $\frac{3}{29}$ Ⓑ $\frac{38}{87}$ Ⓒ $\frac{56}{87}$ Ⓓ $\frac{1}{10}$

38. Ravi has 10 pairs of socks in a drawer, but none of the pairs are matched up. Each pair is a different color, and one pair is blue. Ravi has to pick his socks in the dark so he does not wake his brother. Which expression can be used to find the probability that Ravi will choose a blue sock and then the matching sock?

Ⓕ $\frac{1}{20}+\frac{1}{20}$ Ⓖ $\frac{1}{10}\cdot\frac{1}{20}$ Ⓗ $\frac{1}{20}+\frac{1}{19}$ Ⓙ $\frac{1}{10}\cdot\frac{1}{19}$

CHALLENGE AND EXTEND

39. **Basketball** Terrance has made 90% of the free throws he has attempted at basketball practice. What is the probability that he will make the next three free throws he attempts? **72.9%**

40. A number cube is rolled three times. What is the probability of rolling a 5 at least once? **$\frac{91}{216}$**

Geometry **Use the grid for Exercises 41–44.**

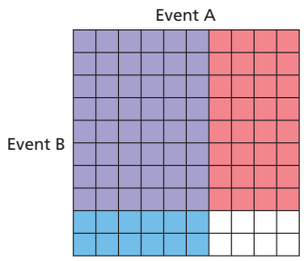
On the grid at right, one small square represents 1% probability.

The shaded rows represent the probability that event *A* occurs.

The shaded columns represent the probability that event *B* occurs.

The area where the shaded rows and columns overlap represents the probability that both events occur.

Use the grid to find each probability.



41. Event *A* occurs. **80%**

42. Event *B* occurs. **60%**

43. Event *A* occurs AND event *B* occurs. **48%**

44. Neither event *A* nor event *B* occurs. **8%**

TEST PREP DOCTOR For **Exercise 37,** students who chose **D** did not change the sample space for the second probability.

Journal

Explain why selecting two objects at the same time can be considered the same as selecting two objects without replacement.

ALTERNATIVE ASSESSMENT

Have students write a set of independent events, explain why they are independent, and then find the probability. Have them repeat the task for dependent events.

Power Presentations with PowerPoint®

10-7 Lesson Quiz

Tell whether each set of events is independent or dependent. Explain your answer.

1. tossing two coins and each coin landing showing heads Independent; the toss of the first coin does not affect the sample space for the toss of the second coin.

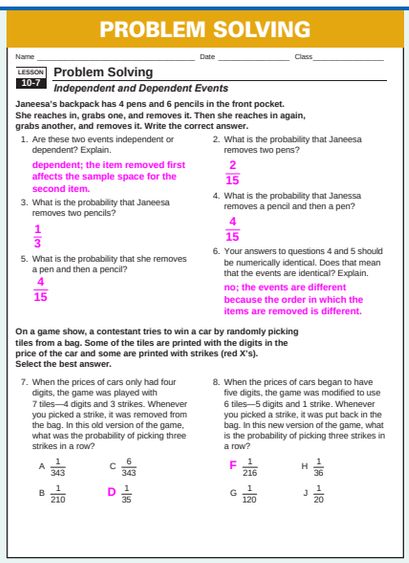
2. drawing a red card from a standard deck of cards and not replacing it; then drawing a black card from the same deck of cards Dependent; there are fewer cards to choose from when drawing the black card.

3. Eight cards are numbered from 1 to 8 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability that both cards are greater than 5? $\frac{3}{28}$

4. An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 3 yellow marbles and 2 white marbles. What is the probability of selecting a white marble and then a yellow marble? $\frac{6}{25}$

5. A number cube is rolled two times. What is the probability of rolling an even number first and then a number less than 3? $\frac{1}{6}$

Also available online



PROBLEM SOLVING

Name _____ Date _____ Class_____

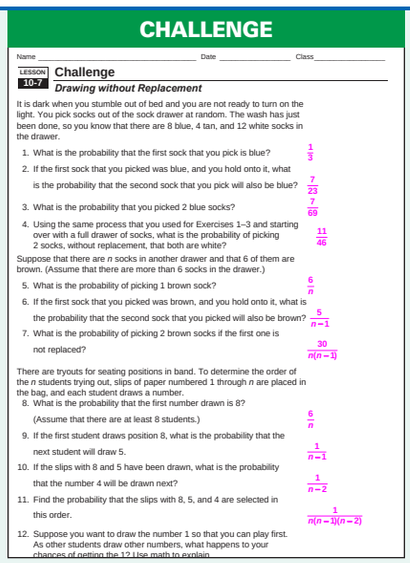
LESSON 10-7 **Problem Solving**
Independent and Dependent Events

Janeesa's backpack has 4 pens and 6 pencils in the front pocket. She reaches in, grabs one, and removes it. Then she reaches in again, grabs another, and removes it. Write the correct answer.

1. Are these two events independent or dependent? Explain.
   dependent; the item removed first affects the sample space for the second item.
2. What is the probability that Janeesa removes two pens?
   $\frac{2}{15}$
3. What is the probability that Janeesa removes two pencils?
   $\frac{1}{3}$
4. What is the probability that Janessa removes a pencil and then a pen?
   $\frac{4}{15}$
5. What is the probability that she removes a pen and then a pencil?
   $\frac{4}{15}$
6. Your answers to questions 4 and 5 should be numerically identical. Does that mean that the events are identical? Explain.
   no; the events are different because the order in which the items are removed is different.

On a game show, a contestant tries to win a car by randomly picking tiles from a bag. Some of the tiles are printed with the digits in the price of the car and some are printed with strikes (red X's). Select the best answer.

7. When the prices of cars only had four digits, the game was played with 7 tiles—4 digits and 3 strikes. Whenever you picked a strike, it was removed from the bag. In this old version of the game, what was the probability of picking three strikes in a row?
   A $\frac{1}{343}$ C $\frac{6}{343}$
   B $\frac{1}{210}$ D $\frac{1}{35}$
8. When the prices of cars began to have five digits, the game was modified to use 6 tiles—5 digits and 1 strike. Whenever you picked a strike, it was put back in the bag. In this new version of the game, what is the probability of picking three strikes in a row?
   F $\frac{1}{216}$ H $\frac{1}{36}$
   G $\frac{1}{120}$ J $\frac{1}{20}$



CHALLENGE

Name _____ Date _____ Class_____

LESSON 10-7 **Challenge**
Drawing without Replacement

It is dark when you stumble out of bed and you are not ready to turn on the light. You pick socks out of the sock drawer at random. The wash has just been done, so you know that there are 8 blue, 4 tan, and 12 white socks in the drawer.

1. What is the probability that the first sock that you pick is blue? $\frac{1}{3}$
2. If the first sock that you picked was blue, and you hold onto it, what is the probability that the second sock that you pick will also be blue? $\frac{7}{23}$
3. What is the probability that you picked 2 blue socks? $\frac{7}{69}$
4. Using the same process that you used for Exercises 1–3 and starting over with a full drawer of socks, what is the probability of picking 2 socks, without replacement, that both are white? $\frac{11}{46}$

Suppose that there are *n* socks in another drawer and that 6 of them are brown. (Assume that there are more than 6 socks in the drawer.)

5. What is the probability of picking 1 brown sock? $\frac{6}{n}$
6. If the first sock that you picked was brown, and you hold onto it, what is the probability that the second sock that you picked will also be brown? $\frac{5}{n-1}$
7. What is the probability of picking 2 brown socks if the first one is not replaced? $\frac{30}{n(n-1)}$

There are tryouts for seating positions in band. To determine the order of the *n* students trying out, slips of paper numbered 1 through *n* are placed in the bag, and each student draws a number.

8. What is the probability that the first number drawn is 8? (Assume that there are at least 8 students.) $\frac{6}{n}$
9. If the first student draws position 8, what is the probability that the next student will draw 5. $\frac{1}{n-1}$
10. If the slips with 8 and 5 have been drawn, what is the probability that the number 4 will be drawn next? $\frac{1}{n-2}$
11. Find the probability that the slips with 8, 5, and 4 are selected in this order. $\frac{1}{n(n-1)(n-2)}$
12. Suppose you want to draw the number 1 so that you can play first. As other students draw other numbers, what happens to your chances of getting the 1? Use math to explain.

Organizer

Use with the lesson *Independent and Dependent Events*

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Find the probability of compound events.

Online Edition
Algebra Tiles

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

Students may find it easier to discover the rules if they do not reduce the fractions. That is, keep a denominator of 36.

The symbol $\cup$ is sometimes used for the word OR and the symbol $\cap$ is sometimes used for the word AND.

Using these symbols, the rule can also be written as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $P(A \cap B) = 0$ for mutually exclusive events.

Look for and express regularity in repeated reasoning.

CC.9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Learn It Online
State Resources Online

10-7 Algebra LAB

Use with Independent and Dependent Events

Compound Events

When two events cannot happen at the same time, they are called *mutually exclusive* events. When two events can happen at the same time, they are called *inclusive* events. You can use sample spaces to determine the probabilities of mutually exclusive events and inclusive events.

When finding the probablity of a compound event, you should first determine whether the simple events involved are dependent or independent.

Activity 1

Look for and express regularity in repeated reasoning. **CC.9-12.S.CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Suppose you are playing Monopoly and are "just visiting" the Jail. If you roll 7 on your next turn, you will land on Community Chest. If you roll 12, you will land on Chance. What is the probability that on your next roll you land on either Community Chest or Chance?

You cannot land on Community Chest and Chance at the same time, so the events are mutually exclusive. You cannot roll 7 and 12 at the same time, so those events are also mutually exclusive.

The table shows some of the totals when rolling two fair dice. Copy and complete the table. Use one color to circle all rolls with a total of 7. Use a second color to circle all rolls with a total of 12.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Try This

1. What is the total number of possible rolls? 36
2. What is the probability that the total will be 7? $\frac{1}{6}$
3. What is the probability that the total will be 12? $\frac{1}{36}$
4. What is the probability that the total will be 7 or 12? $\frac{7}{36}$
5. What do you notice about the probabilities in Problems 2, 3, and 4?

The sum of the probabilities in Problems 2 and 3 is equal to the probability in Problem 4.

Suppose you are three spaces away from Community Chest and eight spaces away from Chance.

6. What is the probability that on your next roll the total will be 3? $\frac{1}{18}$
7. What is the probability that on your next roll the total will be 8? $\frac{5}{36}$
8. What is the probability that on your next roll the total will be 3 or 8? $\frac{7}{36}$
9. Complete the following statement:

   The probability that one of two mutually exclusive events will occur is the ___?___ of the probabilities of the individual events. sum

In this lab, you discovered this rule: If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Teacher to Teacher

You can make this activity come alive by having students in groups find the probabilities while playing the actual game.

Students can find theoretical probabilities and compare them with the actual ratios of the number of times certain events occurred.

Students will learn to identify the types of probabilities and determine how to find them.

Dr. Katherine Staltare
Yonkers, NY

Activity 2

Suppose you are playing Yahtzee and on the last roll of your turn, you roll two of the five dice. You need to roll either a 1 or a 5 to make what is called a "small straight" (four consecutive numbers).

What is the probability that you will make a small straight?

You can roll a 1 and a 5 at the same time on different dice. These events are inclusive.

(If you roll a 1 and a 5, you will need to look at only four of the dice to make a small straight.)

There are three outcomes that involve rolling a 1 or rolling a 5.

Case 1	Case 2	Case 3
At least one 1	At least one 5	Both a 1 and a 5

Try This

10. Find the probability of Case 1, rolling at least one 1.

- **a.** What is the probability of rolling ⚀ on the first die and any number on the second die? $\frac{1}{6}$
- **b.** What is the probability of rolling any number on the first die and ⚀ on the second die? $\frac{1}{6}$
- **c.** What is the probability of rolling ⚀ on the first die and ⚀ on the second die? $\frac{1}{36}$
- **d.** Parts **a** and **b** both include rolling ⚀ on the first die and ⚀ on the second die. You need to count this outcome only once, so subtract the probability from part **c** from the sum of parts **a** and **b.** $\frac{11}{36}$

11. Find the probability of Case 2, rolling at least one 5. (*Hint:* Repeat the steps in Problem 10.) $\frac{11}{36}$

12. Find the probability of Case 3, rolling a 1 and a 5. $\frac{1}{18}$

13. The probability of rolling a small straight, is the probability of rolling ⚀ *or* ⚄.

Subtract the probability of rolling both ⚀ *and* ⚄ from the sum of the probabilities of rolling ⚀ on one of the dice (Problem 10) and rolling ⚄ on one of the dice (Problem 11). $\frac{5}{9}$

14. Complete the following statement:

The probability that one of two inclusive events will occur is the ___?___ of the probabilities of the individual events ___?___ the probability of the intersection of the two events. **sum; minus**

In this lab, you discovered this rule:

If A and B are inclusive events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Sam Dudgeon/HMH

Close

Key Concept

To find the probability of a compound event, add the individual probabilities. If the events are mutually exclusive, subtract the probability of both occurring at the same time.

Assessment

Journal Have students explain the difference between mutually exclusive events and inclusive events.

SECTION 10B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Probability

The Games People Play Probability is involved in most board and card games.

1. There are 108 cards in a deck for the game Uno. The table shows the number of each type of card found in an Uno deck. What is the probability of randomly selecting a red card from a complete deck of Uno cards? $\frac{25}{108}$

Type of Card	Number in Deck
Blue cards, 0–9	19
Green cards, 0–9	19
Red cards, 0–9	19
Yellow cards, 0–9	19
Draw Two	8 (2 in each color)
Reverse	8 (2 in each color)
Skip	8 (2 in each color)
Wild	4
Wild Draw Four	4

2. Robert is playing Monopoly. He starts the game on the GO space. He wants to avoid the Income Tax space, which he will land on if he rolls a 4 with the two dice. What is the probability that he will land on the Income Tax space? $\frac{1}{12}$

3. In the game Battleship, each player chooses locations for small ships on a 10-by-10 grid. Each player guesses grid spaces to try to locate the other player's ships. A correct guess equals a hit, and by hitting each space occupied by a ship, the ship is sunk. The battleship piece occupies four grid spaces. What is the probability that a player randomly guesses a correct grid space and hits the other player's battleship? $\frac{1}{25}$

(t)Sam Dudgeon/HMH; (b)© Handout/ Hasbro/ Ray Stubblebine/Reuters/CORBIS

INTERVENTION

Scaffolding Questions

1. How many possible outcomes are there when rolling 2 dice? 36 How many ways can Robert roll a total of 4? 3
2. How many red cards are in a complete deck? 26
3. How many groups of 4 grid spaces are in a straight line? 140
4. How could you draw a diagram to help you find the number of possible cards? Draw a tree diagram.

Extension

In Battleship, the submarine ship piece occupies 3 grid spaces. What is the probability that a player randomly guesses a correct grid space and hits the other player's submarine? $\frac{3}{100}$

Quiz for Lessons 10-5 Through 10-7

10-5 Experimental Probability

An experiment consists of pushing the random select button on a CD player for a disc with five tracks. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Track 1	4
Track 2	6
Track 3	2
Track 4	5
Track 5	3

1. Selecting track 4 **25%**
2. Selecting track 3 **10%**
3. Not selecting track 2 **70%**
4. Selecting an odd numbered track **45%**
5. Selecting one of the first 3 tracks **60%**
6. Ms. Bleakman checks 32 papers and finds 2 with no name. What is the experimental probability that a paper chosen at random will have no name? Ms. Bleakman has the papers of 176 students. Predict the number of papers she will find with no name. **6.25%; 11 papers**

10-6 Theoretical Probability

Find the theoretical probability of each outcome.

7. Landing on an odd number when spinning a spinner with 4 equal spaces marked 1, 4, 6, 8 **25%**
8. Randomly selecting a vowel from all the letters in the word VOWELS $\frac{1}{3}$
9. Picking a red marble out of a bag with 3 green, 5 blue, 2 red, and 6 yellow marbles **12.5%**
10. The probability of winning a certain prize is 5%. What are the odds of winning the prize? **1:19**
11. The odds of choosing a winning ticket are 2:8. What is the probability of choosing a winning ticket? **20%**
12. The probability of snow is 70%. What are the odds of it not snowing? **3:7**

10-7 Independent and Dependent Events

13. A physical education class has 12 boys and 18 girls. Each day, the teacher randomly selects a team captain. Assume that no student is absent. What is the probability that the team captain is a girl two days in a row? **36%**
14. After reading to a kindergarten class at the library, Tobey gives out stickers. He has 9 zoo animal stickers and 16 scratch-n-sniff stickers. If Tobey gives the stickers out at random, what is the probability that the first child gets a scratch-n-sniff sticker and the second child gets a zoo animal sticker? **24%**

READY TO GO ON? SECTION 10B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

READY TO GO ON?
Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 10B: Worksheets	*Ready to Go On? Intervention*, Section 10B: Online	
Lesson 10-5	10-5 Intervention	Activity 10-5	Diagnose and Prescribe Online
Lesson 10-6	10-6 Intervention	Activity 10-6	
Lesson 10-7	10-7 Intervention	Activity 10-7	

YES ENRICH

***Ready to Go On? Enrichment*, Section 10B**
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Countdown

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

Answers

1. outcome
2. interquartile range
3. independent events
4. 2003
5. 14 more boys

CHAPTER 10 Study Guide: Review

Vocabulary

bar graph	**first quartile**	**outcome**
box-and-whisker plot	**frequency**	**outlier**
circle graph	**frequency table**	**prediction**
complement	**histogram**	**probability**
cumulative frequency	**independent events**	**random sample**
dependent events	**interquartile range (IQR)**	**range**
equally likely	**line graph**	**sample space**
event	**mean**	**stem-and-leaf plot**
experiment	**median**	**theoretical probability**
experimental probability	**mode**	**third quartile**
fair	**odds**	**trial**

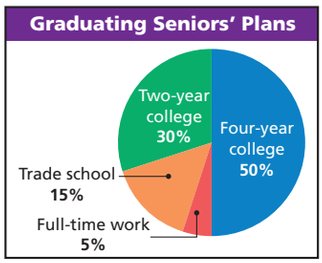
Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is one possible result of an experiment.
2. The ___?___ is the difference between the third and first quartiles.
3. Two events are ___?___ if the occurrence of one event does not affect the probability of the other.

10-1 Organizing and Displaying Data

EXAMPLE

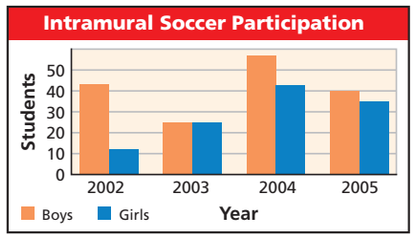
The circle graph shows the post-graduation plans for a high school's 500 graduating seniors.



■ How many seniors plan to attend a four-year college?
50% of 500 *Find the percentage.*
$0.50 \times 500 = 250$
250 students plan to attend a four-year college.

EXERCISES

Use the double-bar graph for Exercises 4 and 5.



4. In which year did the same number of boys and girls participate?
5. How many more boys than girls participated during 2004?

10-2 Frequency and Histograms

EXAMPLE

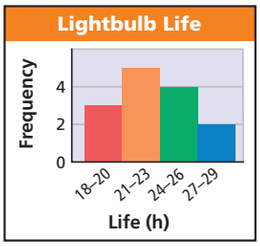
■ **The lives (in hours) of each light bulb in a test are given below. Use the data to make a frequency table and a histogram.**

22, 28, 25, 21, 19, 21, 25,
21, 22, 18, 20, 29, 25, 26

Identify the least and greatest values.
Divide the data into equal intervals.

Light Bulb Life (h)	
Life	Frequency
18–20	3
21–23	5
24–26	4
27–29	2

Draw a bar for the number of hours in each interval. The bars touch, but do not overlap.



EXERCISES

6. The weights of packages shipped by an online retailer are given below. Use the data to make a stem-and-leaf plot.

Package Weight (lb)											
14	9	22	24	7	1	19	22	28	18	12	

7. The numbers of people who attended two different plays are shown in the table below. Make a back-to-back stem-and-leaf plot.

Play Attendance	
Comedy Camp	Days and Days
104 62 83 102 104 120 81 126 122	103 105 80 135 109 128 82 132 139

8. The capacities of the gas tanks on several new vehicles are shown below. Use the data to make a frequency table with intervals.

Gas Tank Capacity (gal)											
15	12	12	15	18	26	25	12	15	18	11	
10	12	16	15	16	18	25	21	18	20	21	

9. Use your frequency table from Exercise 9 to make a histogram.

10-3 Data Distributions

EXAMPLE

■ **Consider the following ages of the winners of an art contest: 13, 15, 14, 18, 12, 10, 11, 13. Find the mean and mode of the data.**

mean:

$$\frac{10 + 11 + 12 + 13 + 13 + 14 + 15 + 18}{8}$$

$$= \frac{106}{8}$$

$$= 13.25$$

mode: 13 *13 occurs most often.*

EXERCISES

Find the mean, median, mode, and range of each data set.

10. Years of playing experience of the members of a musical ensemble:

5, 14, 25, 7, 8, 10, 12, 33, 12

Herman has five pairs of cowboy boots. The prices were $120, $137, $120, $145, and $482.
mean: $200.80 median: $137 mode: $120

11. Which value best describes the price Herman paid? Explain.

Use the data to make a box-and-whisker plot.

12. 25, 28, 2, 24, 28, 21, 18, 29, 31, 12, 6, 19, 27, 3

Answers

6.

Stem	Leaves
0	1 7 9
1	2 4 8 9
2	2 2 4 8

Key: 1|2 means 12

7.

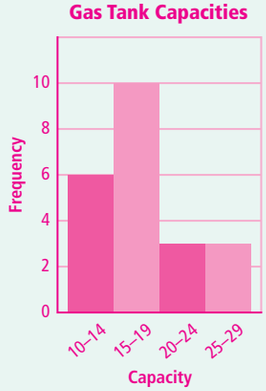
Comedy Camp		Days and Days
2	6	
	7	
3 1	8	0 2
	9	
4 4 2	10	3 5 9
	11	
6 2 0	12	8
	13	2 5 9

Key: |12|8 means 128
4|10| means 104

8.

Gas Tank Capacities		
Capacity	Tally	Frequency
10 – 14	𝍸 I	6
15 – 19	𝍸 𝍸	10
20 – 24	III	3
25 – 29	III	3

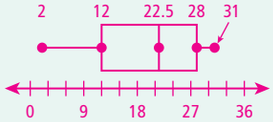
9.



10. mean: 14; median: 12; mode: 12; range: 28

11. Median; the mean is higher than 4 of the 5 prices; the mode is the lowest price.

12.



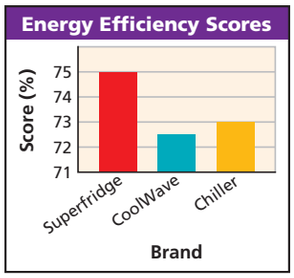
Answers

13. The scale on the vertical axis is too large. This makes the slopes of the segments less steep.
14. Someone might believe that the price has been relatively stable, when in fact it has doubled.
15. 99.5%
16. 24,875
17. 250
18. $\frac{1}{2}$
19. $\frac{1}{4}$
20. $\frac{5}{8}$

10-4 Misleading Graphs and Statistics

EXAMPLE

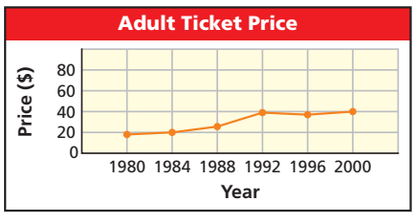
■ **Explain why the graph is misleading.**



The vertical axis begins at 71. This exaggerates the difference in the heights of the bars.

EXERCISES

The graph shows the cost of admission to an amusement park over 20 years.



13. Explain why the graph is misleading.
14. What might someone believe because of the graph?

10-5 Experimental Probability

EXAMPLES

The manager of a photo processing lab inspects 500 photos and finds that 4 have flaws.

■ What is the experimental probability that a photo is flawed?

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{4}{500} = 0.8\%$$

■ In one month, the lab processes 13,000 photos. Predict the number that are likely to be flawed.

$0.008 \cdot 13{,}000 = 104$ *Find 0.8% of 13,000.*

104 photos are likely to be flawed.

EXERCISES

A manufacturer inspects 800 batteries and finds that 796 have no defects.

15. What is the experimental probability that a battery chosen at random has no defects?
16. There are 25,000 batteries in storage. How many batteries are likely to have no defects?
17. Another storage area holds 50,000 batteries. How many batteries are likely to have a defect?

10-6 Theoretical Probability

EXAMPLE

■ **A jar contains red, green, brown, and blue marbles. The probability of choosing red is 0.30, of choosing green is 0.20, and of choosing brown is 0.25. Find the probability of choosing blue.**

$$P(\text{blue}) + P(\text{not blue}) = 1$$
$$P(\text{blue}) + P(\text{red, green, or brown}) = 1$$
$$P(\text{blue}) + (0.30 + 0.20 + 0.25) = 1$$
$$P(\text{blue}) + 0.75 = 1$$
$$P(\text{blue}) = 0.25$$

EXERCISES

Find the theoretical probability of each outcome.

18. Rolling a number less than 4 on a standard number cube
19. Randomly selecting a month that starts with "J" from all month names
20. Randomly selecting a vowel from the letters in EQUATION

10-7 Independent and Dependent Events

EXAMPLES

A hardware store shelf holds 12 cans of red paint, 4 cans of yellow paint, and 6 cans of black paint.

- Syd selects one can at random and replaces it. Then she selects another can at random. What is the probability that Syd selects a red can and then a yellow can?

$$P(\text{red, yellow}) = P(\text{red}) \cdot P(\text{yellow})$$
$$= \frac{12}{22} \cdot \frac{4}{22} \quad \textit{Independent events}$$
$$= \frac{48}{484} = \frac{12}{121}$$

- Gene selects one can at random and then selects another can at random from the remaining cans. What is the probability that Gene selects two cans of black paint?

$$P(\text{black, black}) = P(\text{black}) \cdot P(\text{black after black})$$
$$= \frac{6}{22} \cdot \frac{5}{21} \quad \textit{Dependent events}$$
$$= \frac{30}{462} = \frac{5}{77}$$

EXERCISES

Tell whether each set of events is independent or dependent. Explain your answers.

21. A computer generates a random number and then generates another random number.

22. You roll two number cubes. One is a 6 and the other is a 1.

23. Two audience members are called to the stage.

A lottery machine contains different-colored balls. There are 64 green, 128 yellow, 1 golden, and 3 silver balls. Find the probability of each event.

24. A yellow ball is drawn and set aside. Then a green ball is drawn.

25. A golden ball is drawn and set aside. Then another golden ball is drawn.

26. A green ball is drawn and replaced. Then another green ball is drawn.

Answers

21. The result of the first random number does not affect the result of the second random number, so the events are independent.

22. The result of the first roll does not affect the result of the second, so the events are independent.

23. There are fewer people to choose from after the first person is called, so the events are dependent.

24. $\frac{2048}{9555}$

25. 0

26. $\frac{256}{2401}$

CHAPTER 10

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

Assessment Resources

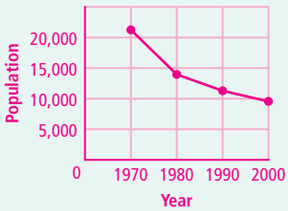
Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

1.

Oakville Population



A line graph is appropriate for this data because it will show the change in population over a period of time.

Learn It Online
State Resources Online

CHAPTER 10

CHAPTER TEST

Population of Oakville	
Year	Population
1970	20,851
1980	14,229
1990	11,198
2000	9,579

1. The table shows the population of Oakville. Use the data to make a graph. Explain why you chose that type of graph.
2. Which ten-year period saw the greatest change in population?
3. Describe the trend in Oakville's population. from 1970 to 1980 The pop. is decreasing, but by smaller amounts as time passes.

The high temperatures in degrees Fahrenheit for two weeks are given:
64, 66, 63, 58, 59, 55, 51, 54, 61, 62, 68, 70, 63, 63.

4. Use the data to make a stem-and-leaf plot.
5. Use the data to make a frequency table with intervals.
6. Use your frequency table from Problem 5 to make a histogram.

The lengths of statements during a town council meeting are given.

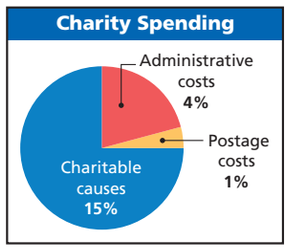
Length of Statement (min)						
6	25	12	14	2	13	38
22	21	14	3	8	5	17

7. Find the mean, median, mode, and range of the data.
8. Use the data to make a box-and-whisker plot.

A manufacturer inspects 500 watches and finds that 498 have no defects.

9. What is the experimental probability that a watch chosen at random has no defects? 99.6%
10. There are 30,000 watches in a warehouse. Predict the number of watches that are likely to have no defects. 29,880

The graph shows how the money raised by a charity is spent.



11. Explain why the graph is misleading.
12. What might someone believe because of the graph?
13. Who might want to use this graph?
14. A bag contains 12 cards, each with a different month of the year printed on it. A card is selected at random. What is the probability that the month begins with *A*? $\frac{1}{6}$
15. The odds of spinning red on a spinner are 2:7. What is the probability of not spinning red? $\frac{7}{9}$
16. A bag has 14 red marbles and 10 white marbles. Rosa randomly picks two marbles from the bag, one at a time. What is the probability that Rosa picks two white marbles? $\frac{15}{92}$

4. High Temperatures for Two Weeks

Stem	Leaves
5	1 4 5 8 9
6	1 2 3 3 3 4 6 8
7	0

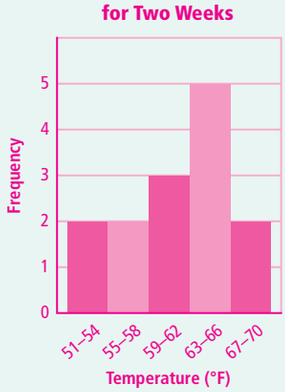
Key: 5|1 means 51

5.

High Temperature for Two Weeks		
Temperatures (°F)	Tally	Frequency
51–54	II	2
55–58	II	2
59–62	III	3
63–66	IIII	5
67–70	II	2

6.

High Temperature for Two Weeks



7, 8, 11–13. See Additional Answers.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 10

FOCUS ON SAT MATHEMATICS SUBJECT TEST

There are two levels of SAT Subject Tests: Mathematics Level 1 and Mathematics Level 2. Each test has 50 multiple-choice questions. The content of each test is very different. Getting a high score on one test does not mean you will get a high score on the other test.

HOT TIP! You can write all over the test book to sketch figures, do scratch work, or cross out incorrect answers to help you eliminate choices. Remember to mark your final answer on the answer sheet because the test book is not used for scoring.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. At a certain high school, the ratio of left-handed to right-handed basketball players is 1:4. If there are a total of 20 players on the team, how many players are right-handed?

(A) 1
(B) 4
(C) 5
(D) 12
(E) 16

2. Your friend randomly selects 2 movies from your collection to borrow. Your collection includes 9 comedies, 5 dramas, 3 mysteries, 1 musical, and 2 horror movies. What is the probability that both movies are comedies?

(A) $\frac{1}{36}$
(B) $\frac{18}{95}$
(C) $\frac{81}{400}$
(D) $\frac{2}{9}$
(E) $\frac{331}{380}$

3. A student's test scores are 88, 87, 82, 95, and 89. If the student needs an average test score of at least 90 to obtain a grade of A, what must the minimum score be on the last test of the semester?

(A) 88
(B) 88.2
(C) 91
(D) 99
(E) 100

4. What is the probability of not getting a 2 on the spinner?

(A) 0.10
(B) 0.25
(C) 0.50
(D) 0.75
(E) 1

5. If $x - 3 = 4 - 2(x + 5)$, then $x =$?

(A) -3
(B) -1
(C) 1
(D) $\frac{3}{2}$
(E) $\frac{11}{3}$

TEST PREP DOCTOR

1. Students who chose **B** found the number of left-handed players. Students who chose **C** found one fourth of 20, not being careful to distinguish the numbers for right and left and also not realizing that the ratio of 1 : 4 means that only 1 out of 5 students in total are left-handed.

2. Students who chose **C** used the formula for independent events instead of the formula for dependent events. Students who chose **A** found the probability of selecting 2 movies from the 9 comedies.

3. Students who chose **A** found the median of the given test scores. Students who chose **B** found the mean of the given test scores.

4. Students who chose **B** found the probability of getting a 2 on the spinner.

5. Students who chose **A** made a mistake in combining the x-terms. Students who chose **C** failed to distribute the 2. Students who chose **D** distributed incorrectly and made a division mistake.

Organizer

Objective: Provide practice for college entrance exams such as the SAT Mathematics Subject Tests.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the SAT Mathematics Subject Tests Levels 1 and 2 represent the following math content areas:

	Level	
	1	2
Number and Operations	10–14%	10–14%
Algebra and Functions	38–42%	48–52%
Geometry and Measurement	38–42%	28–32%
Plane Euclidean	18–22%	0%
Coordinate	8–12%	10–14%
Three-dimensional	4–6%	4–6%
Trigonometry	6–8%	12–16%
Data Analysis, Statistics, and Probability	6–10%	6–10%

Items on this page focus on:

- Data Analysis, Statistics, and Probability

CHAPTER 10 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler focuses on choosing the best answer when there are multiple correct answers or no correct answers. Remind students that they must check *all* of the answers if the answer is "all of the above." Encourage students to combine the strategies for all of the above and none of the above with other strategies they have learned, such as eliminating answer choices and working backward from the answer.

Multiple Choice: *None of the Above* or *All of the Above*

In some multiple-choice test items, one of the options is *None of the above* or *All of the above*. To answer these types of items, first determine whether each of the other options is true or false. If you find that more than one option is true, then the correct choice is likely to be *All of the above*. If none of the options are true, the correct choice is *None of the above*.

If you do not know how to solve the problem and have to guess, *All of the above* is most often correct, and *None of the above* is usually incorrect.

EXAMPLE

There are 8 players on the chess team. Which of the following expressions gives the number of ways in which the coach can choose 2 players to start the game?

(A) ${}_8C_2$ (C) 28

(B) $\frac{8!}{2!(6!)}$ (D) All of the above

Notice that choice D is All of the above. *This means that you must look at each option. As you consider each option, mark it true or false in your test booklet.*

A Because order does not matter, this is a combination problem. The number of combinations of 8 players, taken 2 at a time, is given by ${}_nC_r$, where $n = 8$ and $r = 2$, so ${}_8C_2$ is a correct model of the combination. Choice A is true.

B The number of combinations of 8 players, taken 2 at a time, is given by ${}_nC_r = \frac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!}$$

Choice B is a correct model of the combination. Choice B is also true. The answer is likely to be choice D, *All of the above*, but you should also check whether choice C is true.

C The number of combinations of 8 players, taken 2 at a time, is given by ${}_nC_r = \frac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!} = 28$$

Choice C is a correct model of the combination. Choice C is true as well.

Because A, B, and C are all true, the correct response is D, *All of the above*.

HOT TIP! Be careful of problems that contain more than one negative word, such as *no*, *not*, or *never*. Read the problem and each option twice before selecting an answer.

Read each test item and answer the questions that follow.

Item A

The mean score on a test is 68. Which CANNOT be true?

Ⓐ Every score is 68.

Ⓑ Half of the scores are 68, and half of the scores are 0.

Ⓒ Half of the scores are 94, and half of the scores are 42.

Ⓓ None of the above

1. What is the definition of *mean*?
2. If you find that an option is true, is that the correct response? Explain.
3. Willie determined that A and C could both be true, so he chose D as his response. Do you agree? Why or why not?

Item B

What is the probability of rolling a 2 on a number cube?

Ⓕ $16.\overline{6}\%$

Ⓖ $1 - P(\text{rolling 1, 3, 4, 5, or 6})$

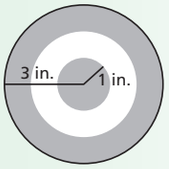
Ⓗ $\frac{1}{6}$

Ⓙ All of the above

4. What is the complement of rolling a 2? Is choice G correct? Explain.
5. If you roll a number cube, how many possible outcomes are there? How does this information help you solve this problem?
6. Is the value given in choice H equivalent to any other choice? If so, which one(s)?
7. How many choices are true? What is the correct response to the test item?

Item C

Suppose that a dart lands at a random point on the circular dartboard. Find the probability that the dart does NOT land inside the center circle.



Ⓐ $\frac{1}{9}$

Ⓑ 8π

Ⓒ $\frac{8}{9}$

Ⓓ None of the above

8. Kyle finds that choices A and B are both false. To save time, he selects choice D as his answer because he figures it is likely that choice C will also be false. Do you think Kyle made a wise decision? Why or why not?
9. What is the formula for the area of a circle? What is the total area of this dartboard? How can you determine the area of the dartboard outside the center circle?
10. Determine whether choices A, B, and C are true, and then give the correct response to this test item.

Item D

Each gym member receives a 3-digit locker combination. Digits are from 0–9 and may repeat. What is the probability that you will receive a code consisting of three identical numbers?

Ⓕ $\frac{1}{100}$

Ⓖ $\frac{10}{{}_{10}C_3}$

Ⓗ ${}_{10}P_3 = \frac{10!}{7!}$

Ⓙ All of the above

11. How can you determine whether choice J is the correct response to the test item?
12. Are the values given in choices F, G, and H equivalent? What does this tell you about choice J?

Answers

Possible answers:

1. the sum of all the values in the set divided by the number of values in the set
2. No; the question asks which statements cannot be true.
3. No; "None of the above" would mean that all of the statements must be true, but choice **B** is not true.
4. Not rolling a 2, or rolling a 1, 3, 4, 5, or 6; yes; the probability of an event is 1 minus the probability of the complement of that event.
5. 6; the probability of rolling a 2 will be 1 outcome out of 6, so choice **H** is correct.
6. yes; **F**
7. 3 (**F, G,** and **H**); **J**
8. No; choice **C** is correct.
9. $A = \pi r^2$; 9π; area outside center circle $= 9\pi - \pi = 8\pi$
10. Only choice **C** is true; the correct response is **C.**
11. Determine whether or not the values in choices **G, F,** and **H** are equivalent.
12. No; choice **J** cannot be the correct choice.

Answers to Test Items

A. B

B. J

C. C

D. F

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources
Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

CUMULATIVE ASSESSMENT

Multiple Choice

1. Josephine has 9 cousins. Her friend Summer has s more cousins. Which expression can be used to show the number of cousins Summer has?

Ⓐ $9 \div s$ Ⓒ $9 + s$
Ⓑ $9s$ Ⓓ $9 - s$

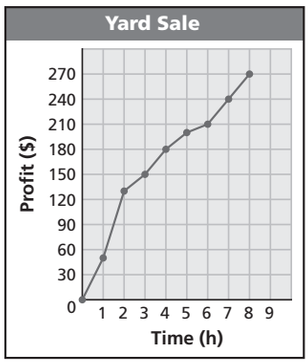
2. A negative number raised to an exponent is positive. Which of the following is NOT true?

Ⓕ The number could be even.
Ⓖ The number could be odd.
Ⓗ The exponent could be even.
Ⓙ The exponent could be odd.

3. A computer game normally sells for $40. Niko bought it when it went on sale for 25% off. The sales tax was 6%. How much did Niko pay for the game?

Ⓐ $10.60 Ⓒ $31.80
Ⓑ $12.40 Ⓓ $32.40

4. The graph shows the profit made at a family yard sale that lasted 8 hours. Between which hours did the profit increase at the slowest rate?



Ⓕ 0–1 Ⓗ 5–6
Ⓖ 1–2 Ⓙ 7–8

5. Which of the following is one of the five values needed to make a box-and-whisker plot?

Ⓐ Mean Ⓒ Mode
Ⓑ Median Ⓓ Average

6. Which value is a solution of $2(1 - x) < 12$?

Ⓕ -7 Ⓗ -5
Ⓖ -6 Ⓙ -4

7. What is the median of the data presented in this stem-and-leaf plot?

Stem	Leaves
6	0 2 2 2 5 8
7	3 4 4 6 7
8	2 5 8 9

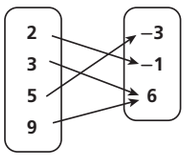
Key: 8 | 2 means 0.82

Ⓐ 0.62
Ⓑ 0.74
Ⓒ 6.2
Ⓓ 7.4

8. Which of the following does NOT represent a function?

Ⓕ $\{(-4, 3), (-2, 1), (-2, -1), (-4, -3)\}$

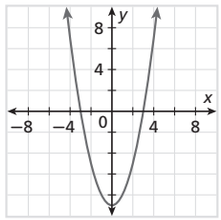
Ⓖ



Ⓗ

x	−4	−3	−2	−1	0	1
y	4	3	2	1	0	−1

Ⓙ



Answers

1. C
2. J
3. C
4. H
5. B
6. J
7. B
8. F
9. B
10. J
11. D
12. G
13. 2
14. 0.64
15. 819.2
16a. {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}
b. $\frac{3}{8} = 37.5\%$

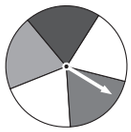
HOT TIP! **If you are allowed to write in your test booklet, you may want to add additional information to a given diagram. Be sure to mark your answer on the answer sheet since your marks on the test booklet will not be graded.**

9. Which of these equations describes a line parallel to $x + 2y = 6$?

Ⓐ $y = \frac{1}{2}x$ Ⓒ $y = -2x + 6$

Ⓑ $y = -\frac{1}{2}x + 4$ Ⓓ $y = 2x + 3$

10. What is the probability that the spinner will NOT land on white?



Ⓕ 20%

Ⓖ 40%

Ⓗ 50%

Ⓙ 60%

11. Which equation is NOT true?

Ⓐ $55 + 27 + 45 = 100 + 27$

Ⓑ $5 \cdot 7 \cdot \frac{2}{5} = 2 \cdot 7$

Ⓒ $14(126) = 14(100) + 14(26)$

Ⓓ $31(152) = 30(150) + 1(2)$

12. Which is a factor of $4x^2 + 24x + 36$?

Ⓕ $x + 12$

Ⓖ $2x + 6$

Ⓗ $2x + 18$

Ⓙ $4x + 18$

Gridded Response

13. Yvette is choosing between two stores to get her family pictures taken. Say Cheese charges a $20 sitting fee and $14 per sheet of pictures. Picture Me charges a $12 sitting fee and $18 per sheet of pictures. For how many sheets will the total cost be the same at both stores?

14. Brian plays basketball. In the past, he has made 4 out of every 5 free throws. What is the probability, as a decimal, that Brian will make the next two free throws?

15. What is the next term in the geometric sequence 2000, 1600, 1280, 1024,...?

Short Response

16. An experiment consists of tossing 3 coins at the same time.

a. Identify the sample space.

b. What is the probability of tossing 2 tails and 1 head?

c. Which is more likely to occur: tossing exactly 1 tail or tossing at least 2 heads? Explain.

17. If 8% of a number is 4, what is 12% of the number? Explain your reasoning.

18. Nat surveyed 30 twelfth-grade students and found that 27 of them were in favor of allowing seniors to leave campus for lunch. As a result, Nat reported that "90% of the student body supports off-campus lunch privileges for seniors." Was Nat's statistic misleading? Explain.

19. As part of a challenge problem, a math teacher writes the following expression on the board:

$$-(-x).$$

a. If x is 12, what is the value of the expression?

b. If x is a negative number, is the value of the expression positive or negative? Explain how you found your answer.

c. Simplify the expression.

Extended Response

20. The ages of the first 15 people to enter a community center for bingo one evening are 77, 85, 76, 73, 69, 70, 84, 82, 76, 74, 89, 83, 68, 91, and 88.

a. Find the mean, median, mode, and range of the data.

b. Create a box-and-whisker plot of the data.

c. The sixteenth person to enter the center is 39 years old. Calculate the mean, median, mode, and range of the data with the 16 people.

d. Create a box-and-whisker plot with the 16 people.

e. When the sixteenth person was added, which measure of central tendency changed the most? Which measure changed the least? Explain.

f. How did the addition of the sixteenth person affect the box-and-whisker plot?

Short-Response Rubric

Items 16–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 20

4 Points = The student correctly answers parts **a** and **c** and draws correct graphs in parts **b** and **d.** The student provides explanations in parts **e** and **f.**

3 Points = The student correctly answers parts **a** and **c** and draws correct graphs in parts **b** and **d** but does not provide explanations in parts **e** and **f.**

2 Points = The student correctly answers parts **a** and **c.** The student draws incorrect graphs in parts **b** and **d** or gives an incorrect explanation in parts **e** or **f.**

1 Point = The student incorrectly answers parts **a** and **c** but draws correct graphs from that information in parts **b** and **d,** or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

16c. at least 2 heads; P(exactly 1 tail) $= \frac{3}{8} = 37.5\%$; P(at least 2 heads) $= \frac{4}{8} = 50\%$

17. 6

18. The sample is biased and small. Nat surveyed only 30 students out of the entire student body. Also, those surveyed were seniors, who are naturally going to be in favor of an action that benefits seniors.

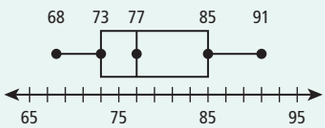
19a. 12

b. negative; because x is negative, $-x$ is positive, and so $-(-x)$ is negative.

c. x

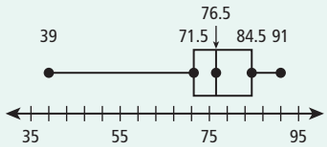
20a. mean: 79; median: 77; mode: 76; range: 23

b. Bingo Ages, 15 People



c. mean: 76.5, median: 76.5, mode: 76, range: 52

d. Bingo Ages, 16 People



e. Mean; mode; the mean decreased because the 16th person was much younger than the other people. The mode remained unchanged.

f. stretched out the left "whisker"

Real-World CONNECTIONS

Organizer

Objective: Choose appropriate problem-solving strategies and use them to solve real-world problems.

INTERACTIVE **Online Edition**

Roadside America Indoor Miniature Village

Reading Strategies

In **Problem 1,** ask students what measurement is represented by the 180 one-inch cubes. the volume
Remind students that the base of a geometric figure is the bottom face.

Using Data Have students find how many square miles of land are shown in the grid.

Learn It Online
State Resources Online

Real-World CONNECTIONS

Roadside America Indoor Miniature Village

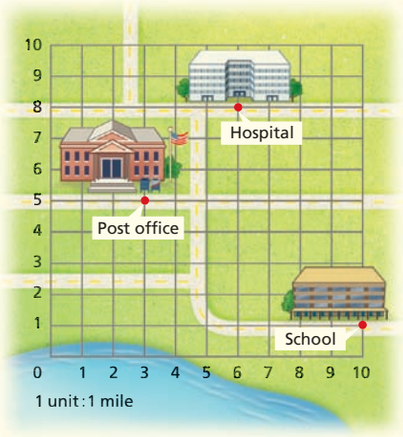
The Roadside America Indoor Miniature Village in Shartlesville, Pennsylvania, is sometimes referred to as the World's Greatest Indoor Miniature Village. The exhibit uses miniature buildings and realistic landscaping to show what life was like in the 1700s, 1800s, and 1900s. Laurence Gieringer built the village using a $\frac{3}{8}$-inch-to-1-foot scale.

Choose one or more strategies to solve each problem.

1. Suppose a model of a building is made of 180 one-inch cubes. The area of the base of the model is 15 in^2. How tall is the miniature building? Using a scale of $\frac{3}{8}$ inch to 1 foot, how tall is the actual building? **12 in.; 32 ft**
2. There are about 4000 miniature people in the village. This number is about 100 more than thirteen times the number of buildings in the village. Estimate the number of miniature buildings in the village. **300**

For 3, use the map.

3. It might be helpful to use a coordinate grid when designing a miniature village. Suppose you decide to arrange it as shown. About how far is it from the school to the post office along a straight line? **about 8 mi**



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Problem-Solving Focus

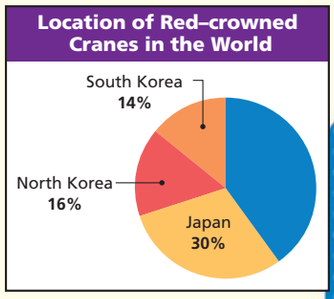
Discuss with students what problem-solving strategies could be used to solve each problem. For example, Use a Model and Work Backward could be used to solve **Problem 1.** Students could build a prism-shaped model of a building with a base that has an area of 15 in^2.

The National Aviary

The National Aviary is located on the north side of Pittsburgh. It is America's only independent indoor nonprofit bird zoo. The aviary is home to more than 600 birds of over 200 species, many of which are threatened or endangered in the wild. Visitors can see birds from almost every continent. Visitors can even feed the birds flying around in a 200,000-cubic-foot indoor rainforest where it actually rains every 15 minutes.

Choose one or more strategies to solve each problem.

For 1 and 2, use the circle graph.



1. The red-crowned crane is the second rarest crane species. The total population of red-crowned cranes in the wild is about 2000 birds. The circle graph shows the approximate percent of this population in four countries where they are found. About how many red-crowned cranes live in Japan? 600

2. About how many of the total population of red-crowned cranes living in the wild do not live in North Korea or South Korea? 1400

3. One exhibit at the aviary contains bald eagles. An eagle can see a rabbit from about one mile away. The average person must be 1650 feet away to see the same rabbit. Do humans or eagles see farther? How many times as far? eagles; 3.2 times as far

4. It is estimated that only one of every 18 attempts by an eagle to hunt for food is successful. Because of this, eagles use a great deal of energy hunting and have to spend a lot of time resting quietly. According to this information, if you observe an eagle diving for a fish, what is the probability that he is not going to catch the fish on the first try? $\frac{17}{18}$

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The National Aviary

Reading Strategies

ENGLISH LANGUAGE LEARNERS

Ask students if they know what *aviators* do. They fly planes. Then ask how this knowledge is related to an *aviary,* which is a large building which houses birds. Both aviators and birds fly.

Remind students that information needed for one problem may be given in a previous problem. Such is the case with **Problem 2.**

Using Data Have students check that the sum of the percents in the circle graph is 100.

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problem 2.** Focus on the second and fourth steps: **(2) Make a Plan:** Ask students for two different ways to solve the problem. You could add the percents for China and Japan and multiply that percent by the total, or you could add the percents for North and South Korea, multiply that percent by the total, and then subtract that number from the total. **(4) Look Back:** Ask students how they can use the circle graph to check their answer for reasonableness. Over half of the shaded area is not for North and South Korea, so the answer should be greater than half of 2000. 1400 is greater than 1000, so the answer is reasonable.